The visibility space

Fundamentals of Radio Interferometry: Chapter 4

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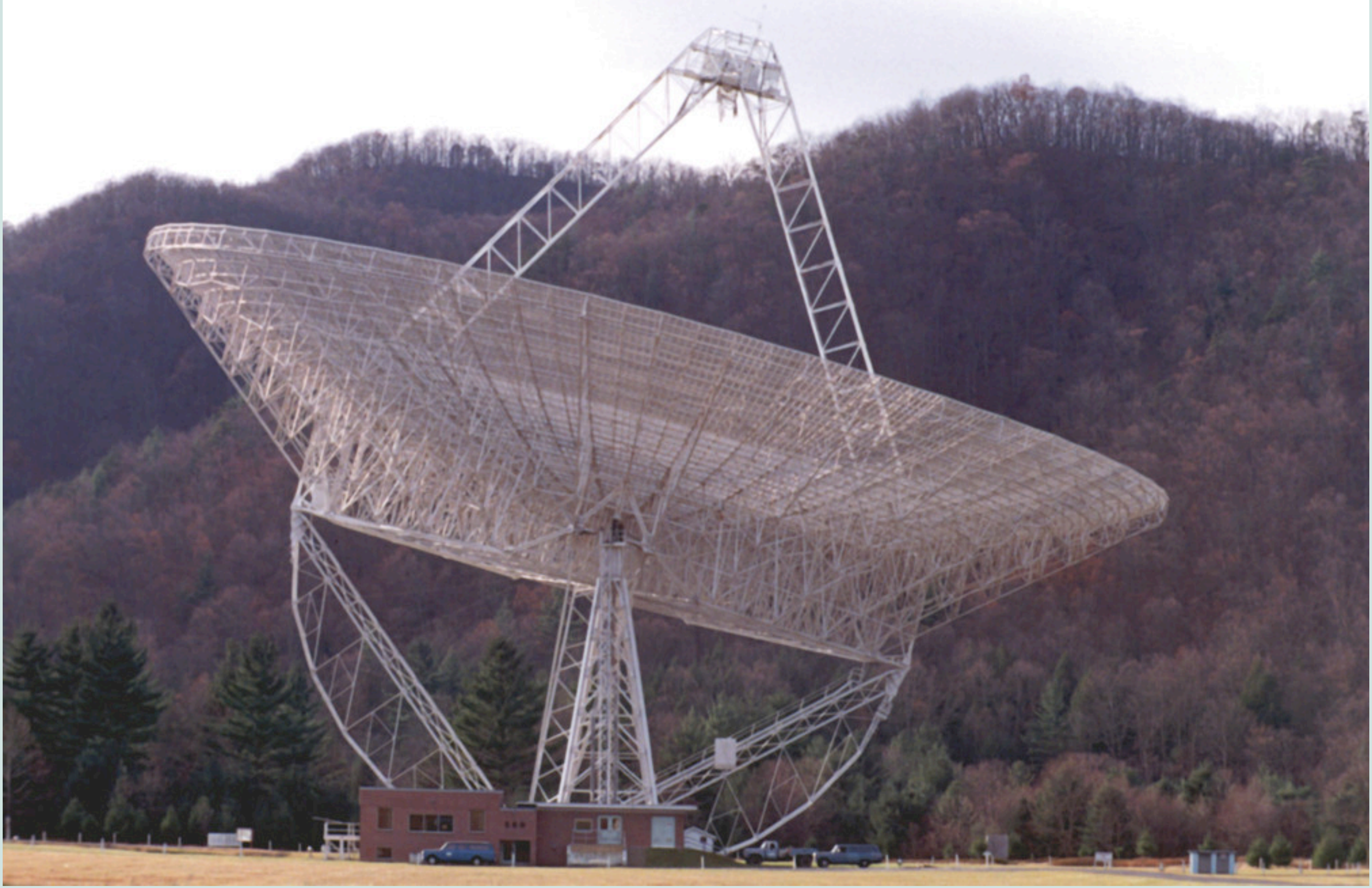
NASSP 2016

Motivation for interferometry : recall

Green Bank Transit Telescope (100 m)



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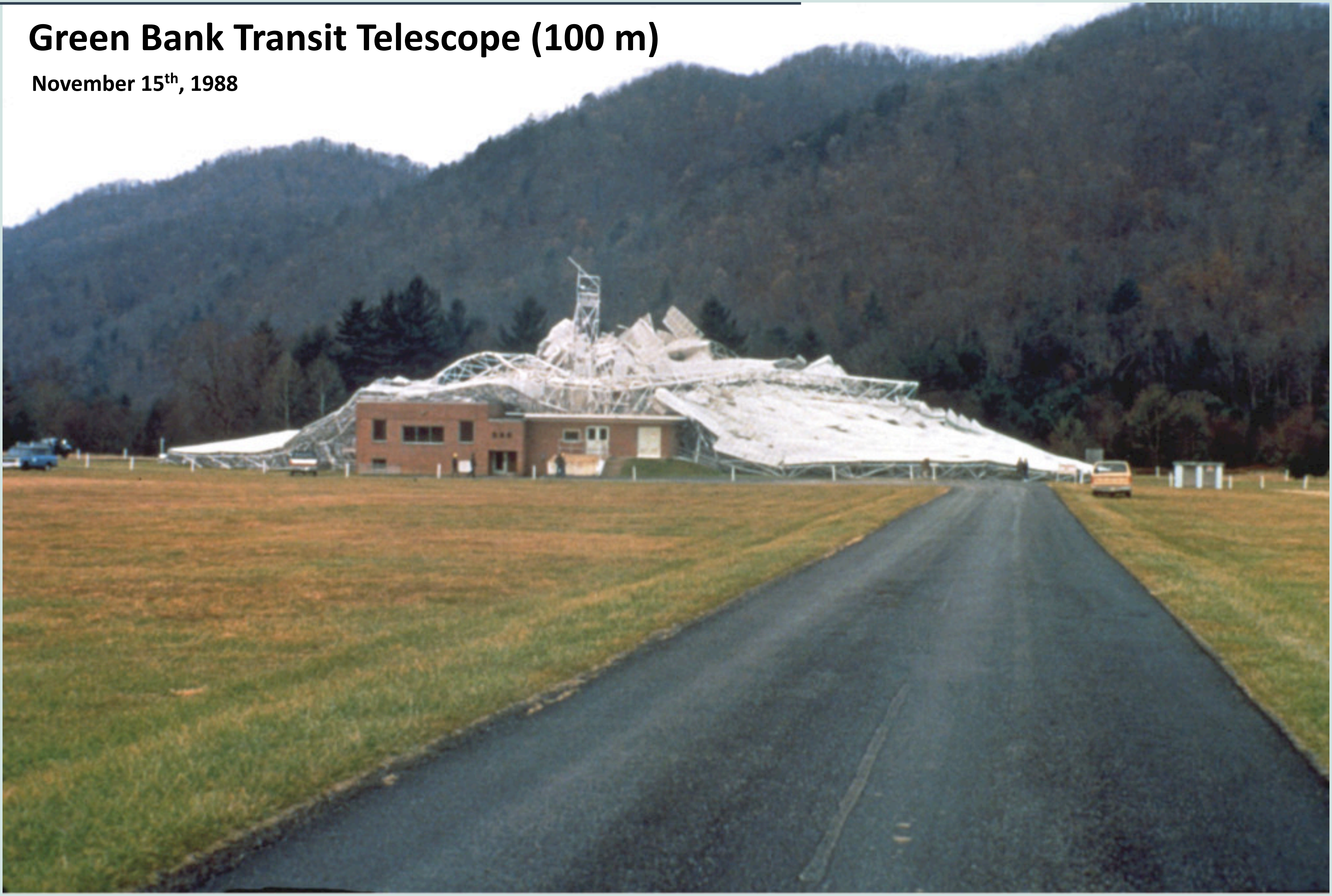

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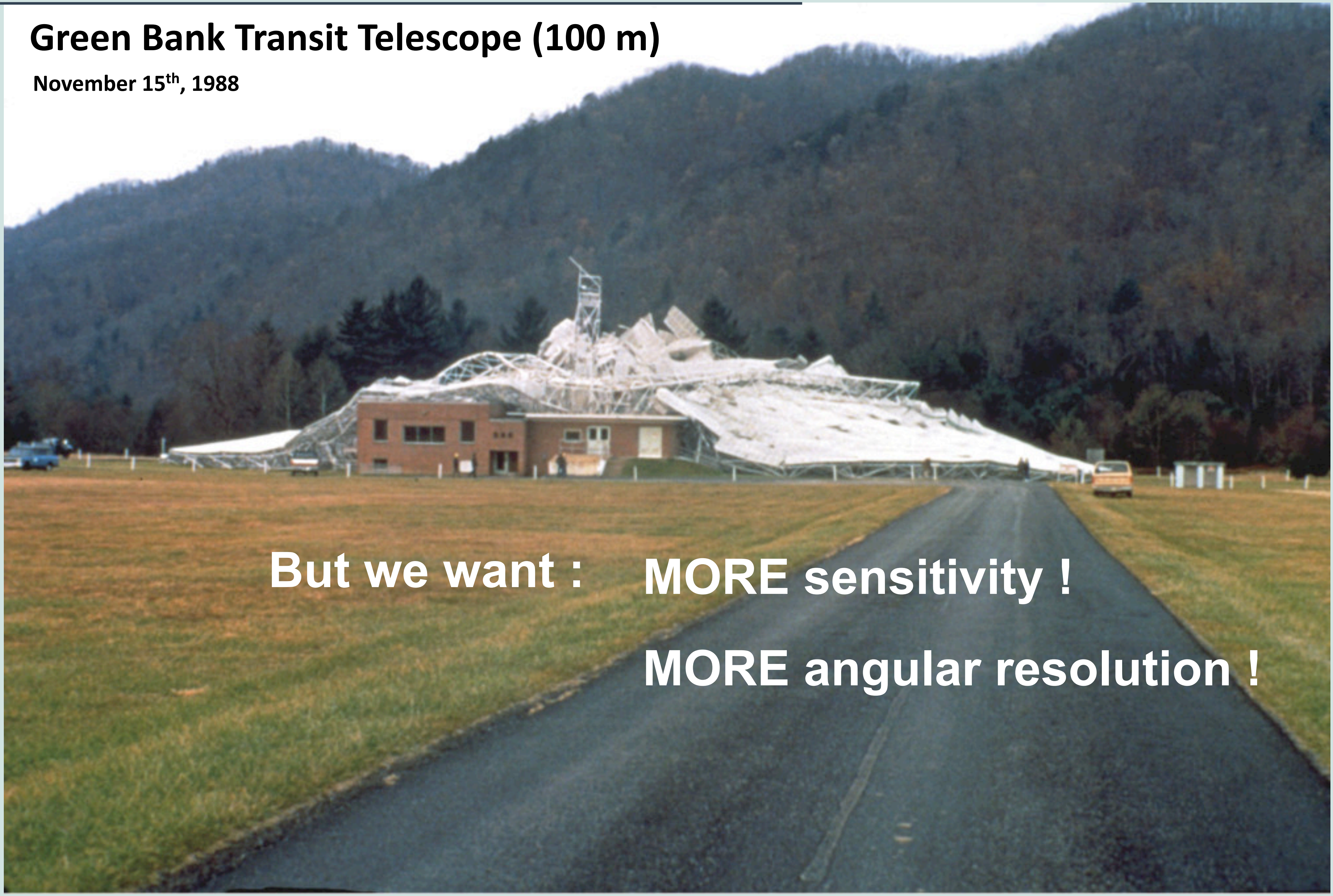
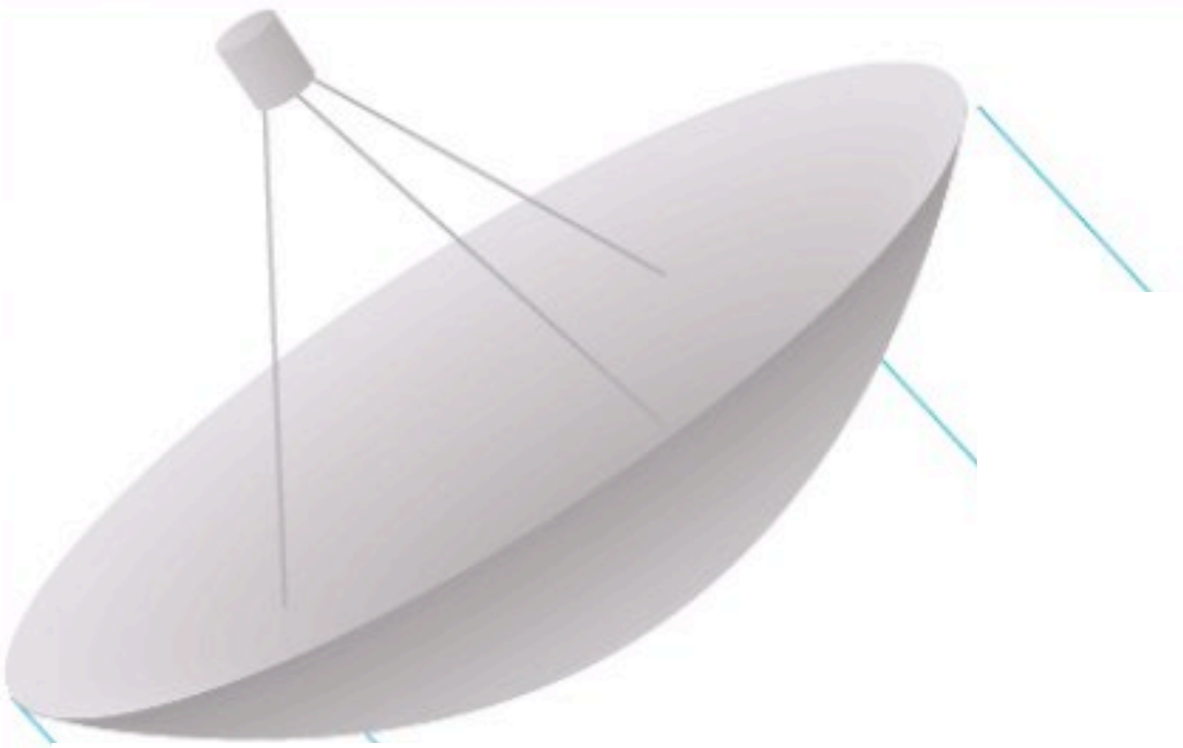


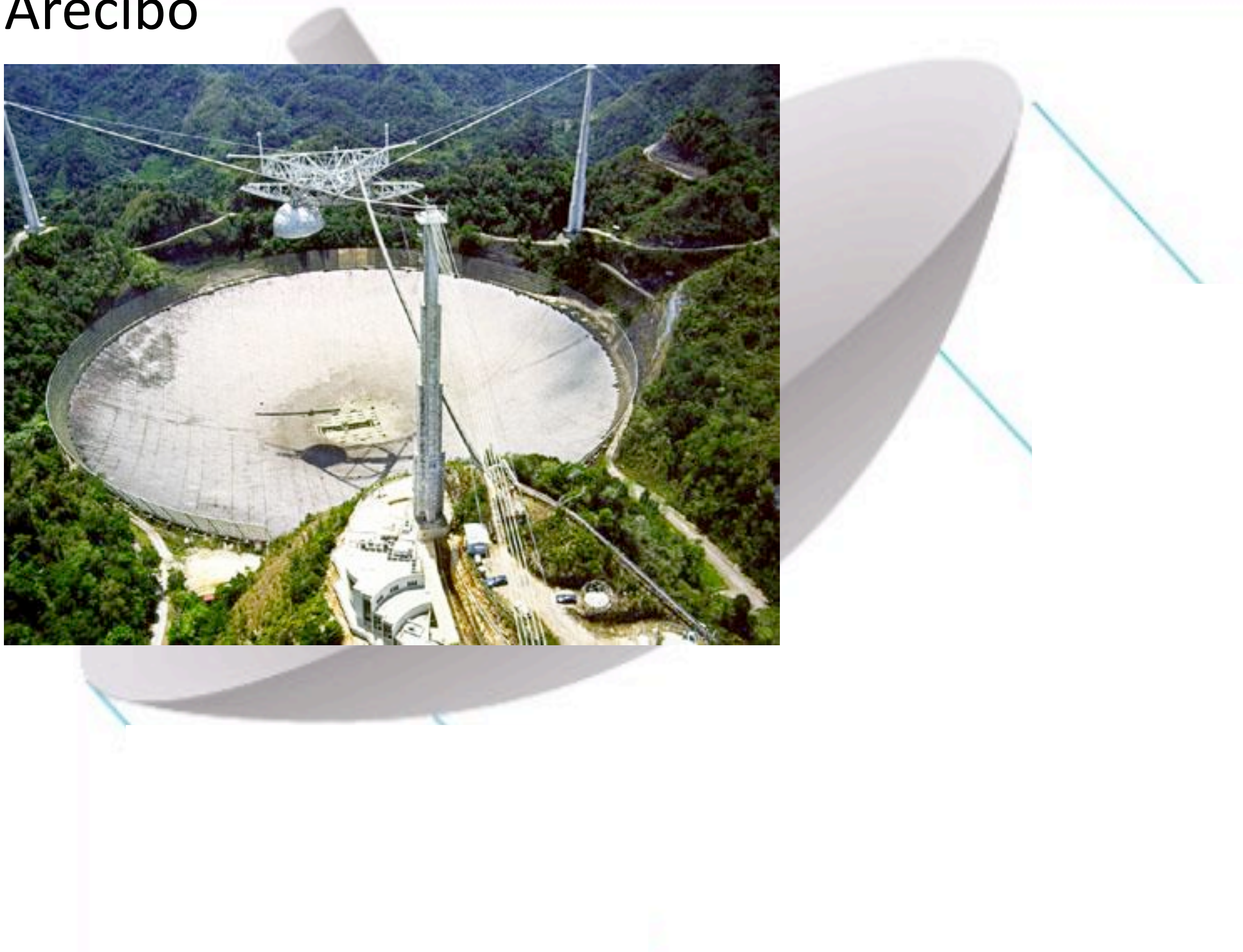
Overview 1: baseline



$$\delta\theta \propto \frac{\lambda}{D}$$

Overview 1: baseline

Arecibo



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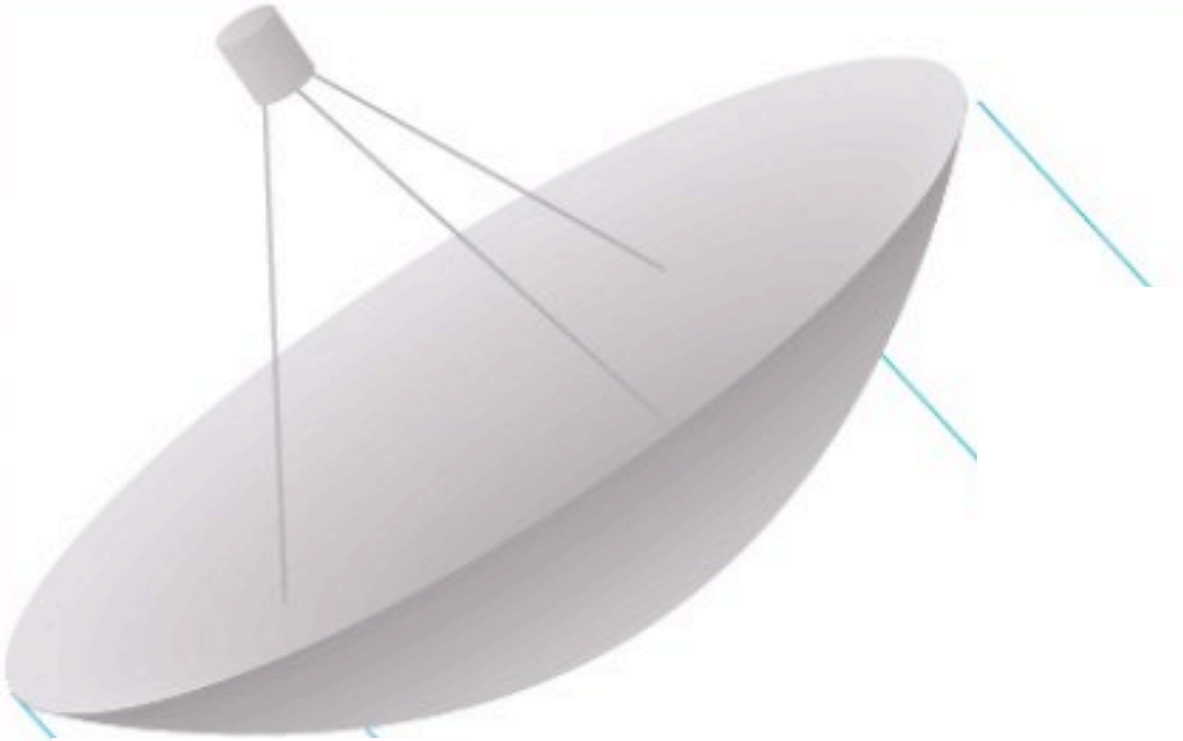

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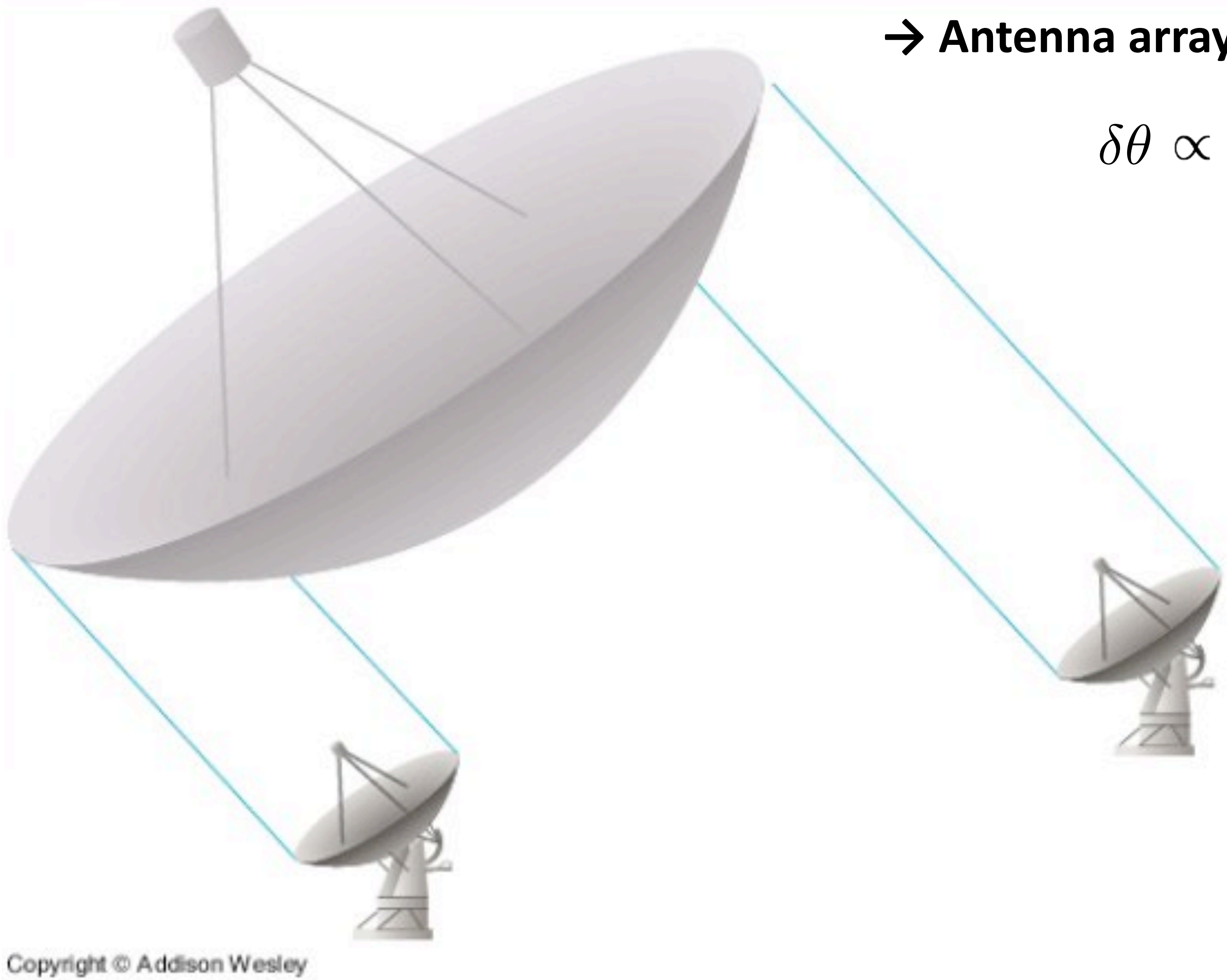

→ Antenna arrays

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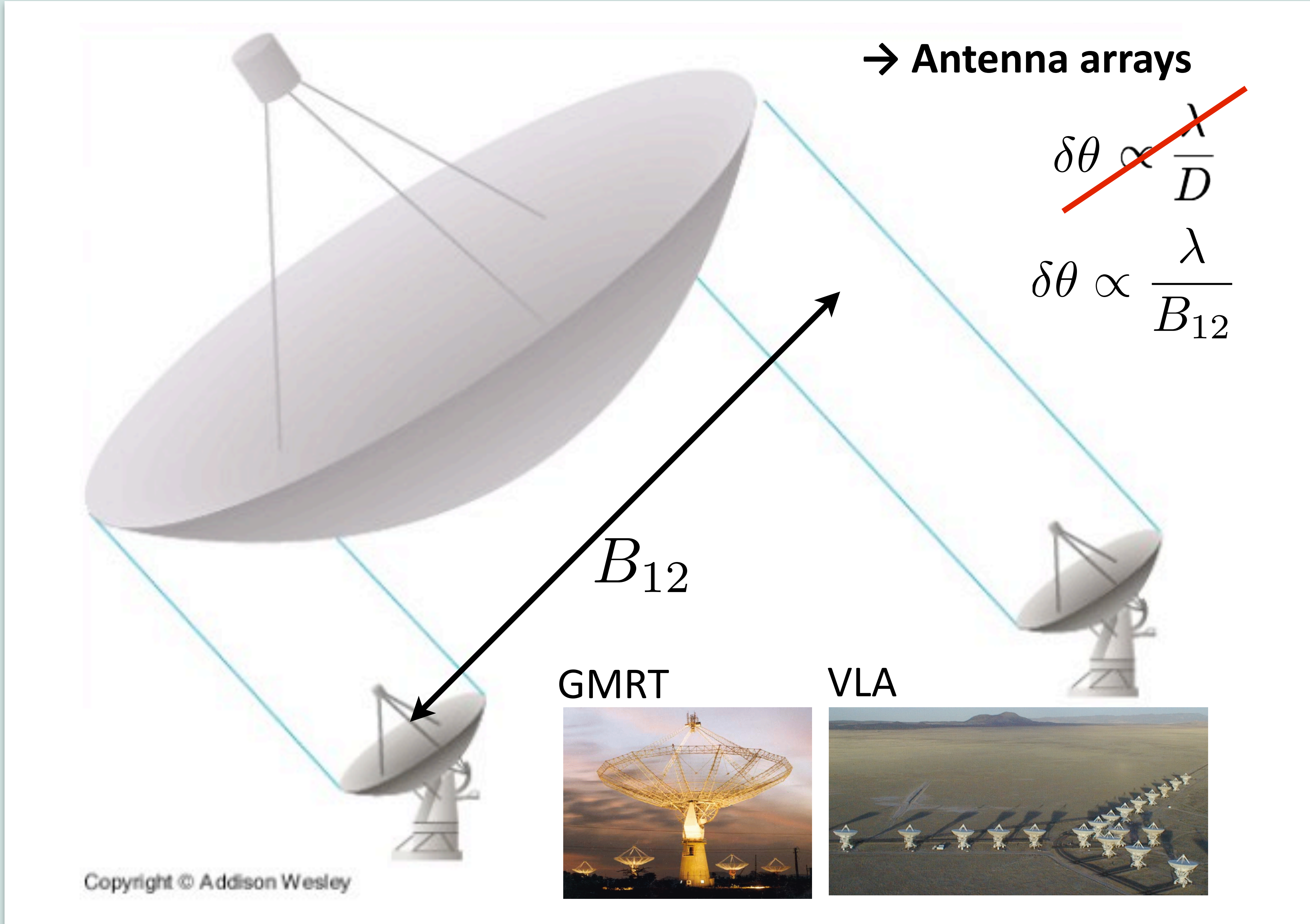

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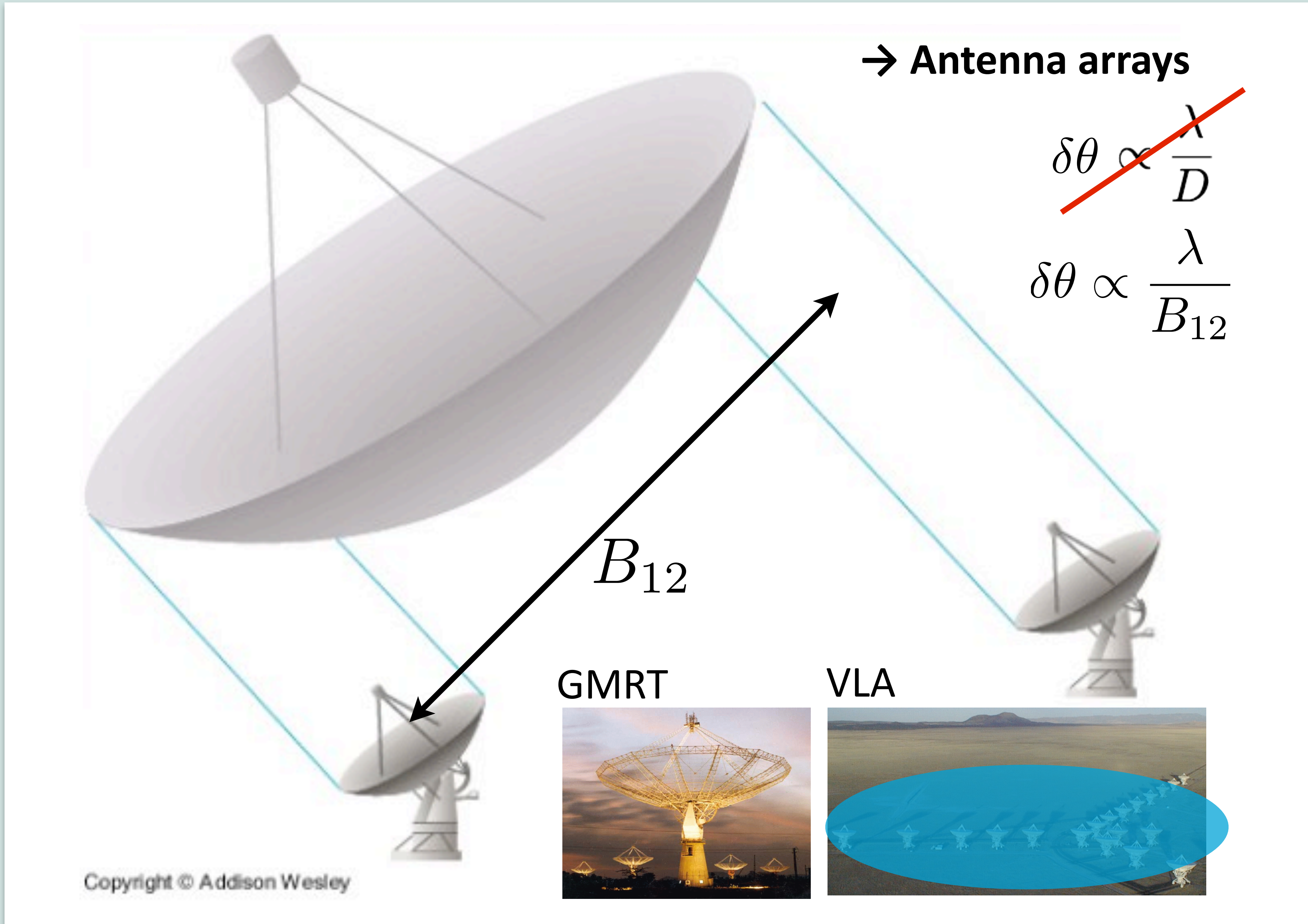
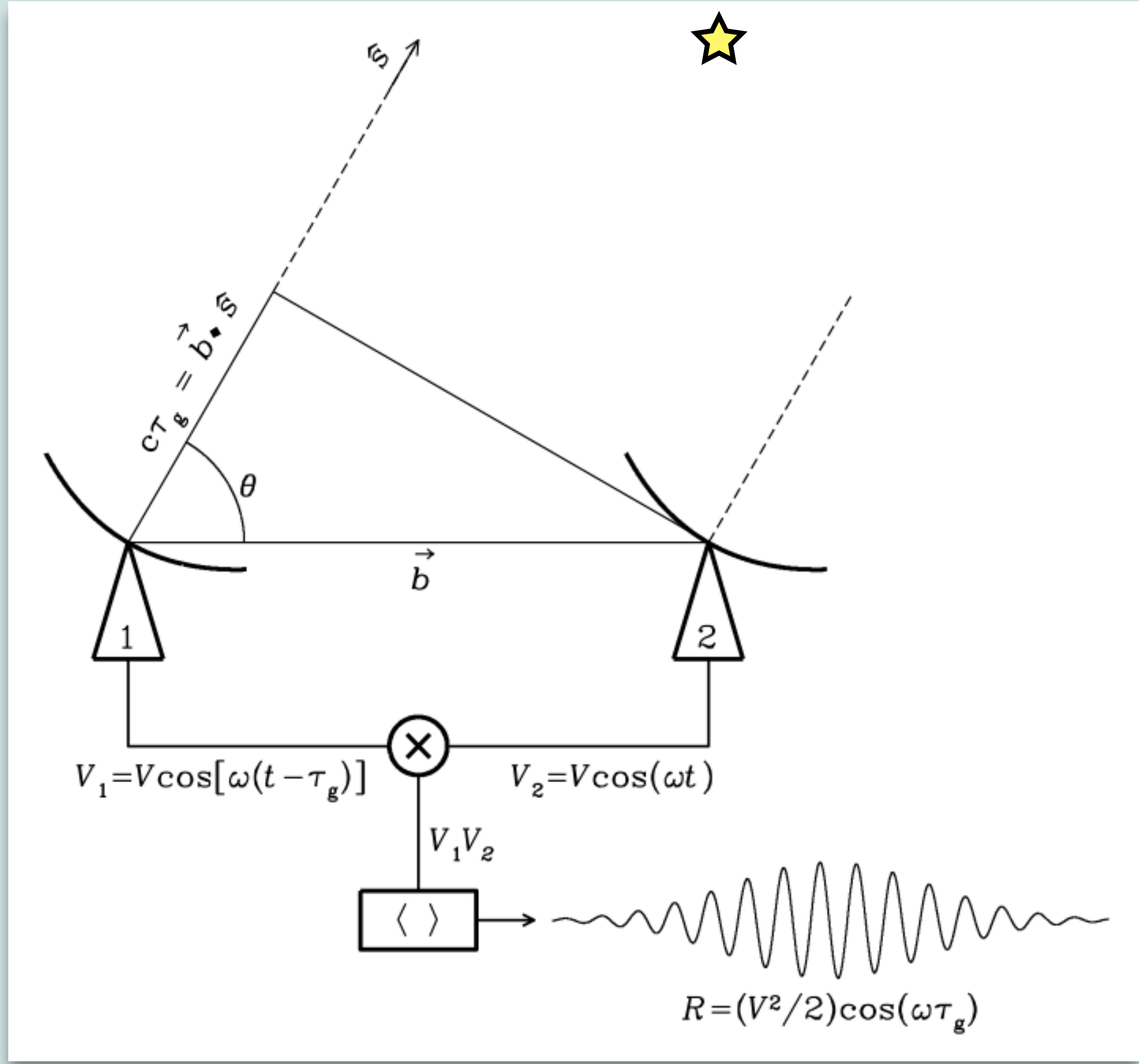


Overview 2: 2-element interferometer



1 baseline b (t,v) + 1 direction s

=

1 spatial frequency of the sky brightness

through the measurement of the **fringe contrast**

or **fringe « visibility»**

$$V_\nu = \int I_\nu(\hat{s}) \exp(-i2\pi\vec{b}\cdot\hat{s}/\lambda)\, d\Omega$$

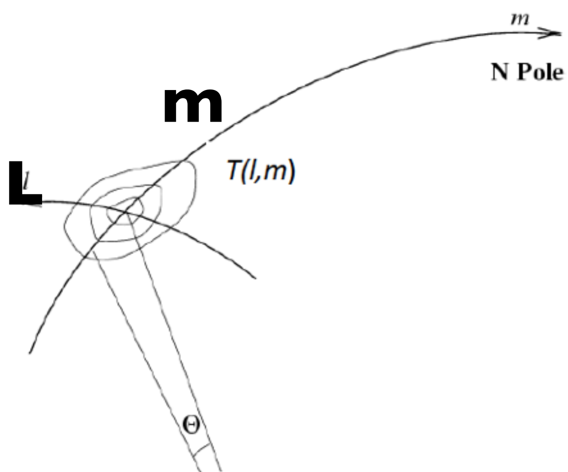
complex visibility $\underline{V}_\nu$

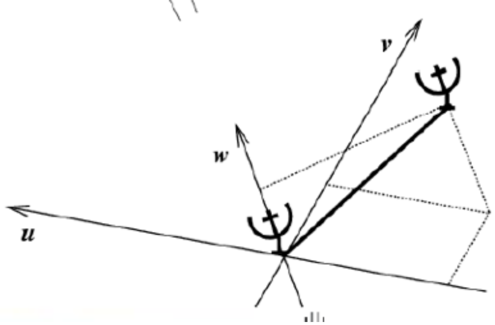
Overview 3: N-element interferometer

N antennas/telescopes

$\dfrac{N(N-1)}{2}$ independent baselines

1 projected baseline
= 1 sample in the Fourier « u,v » plane





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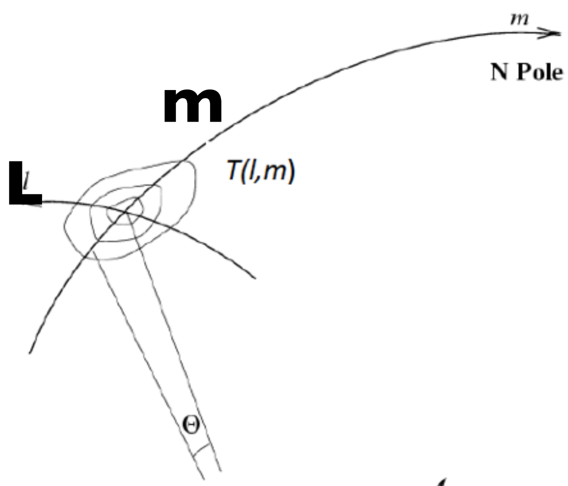

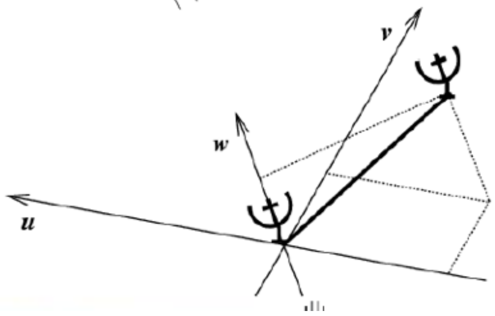

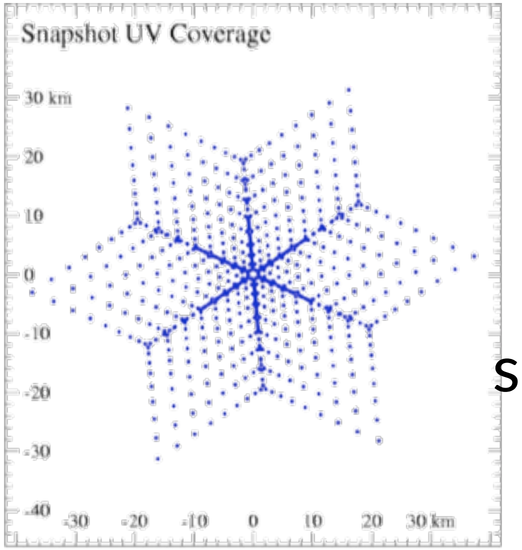

v

u

(u,v) plane sampling

$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm$$

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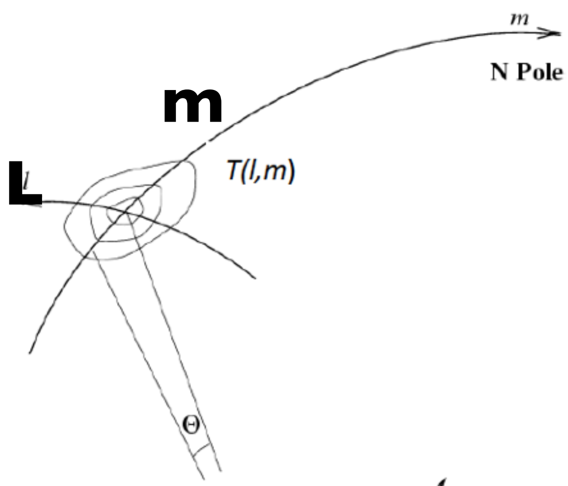

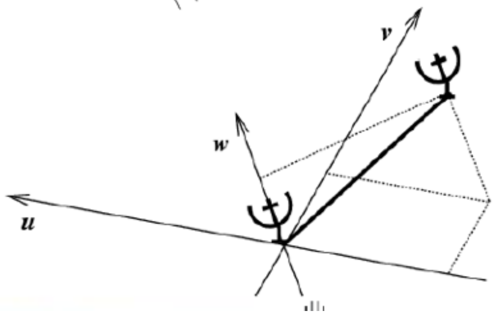

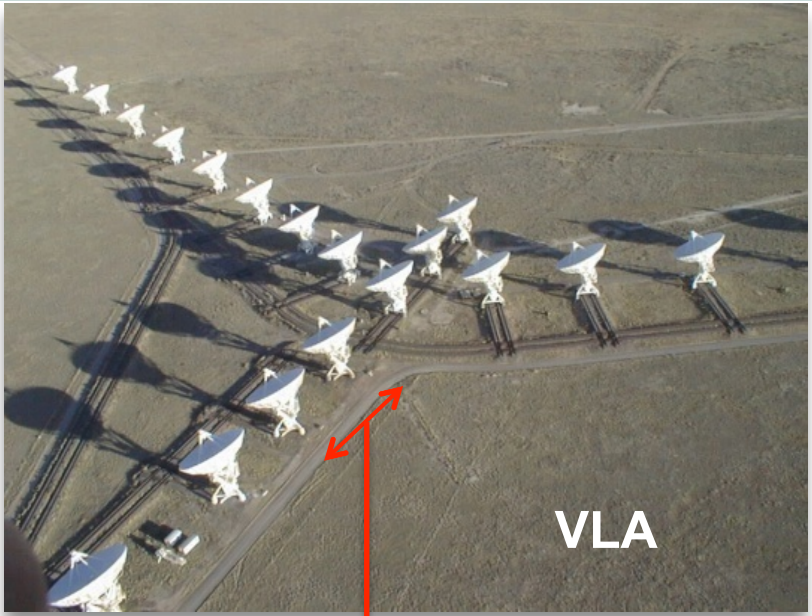

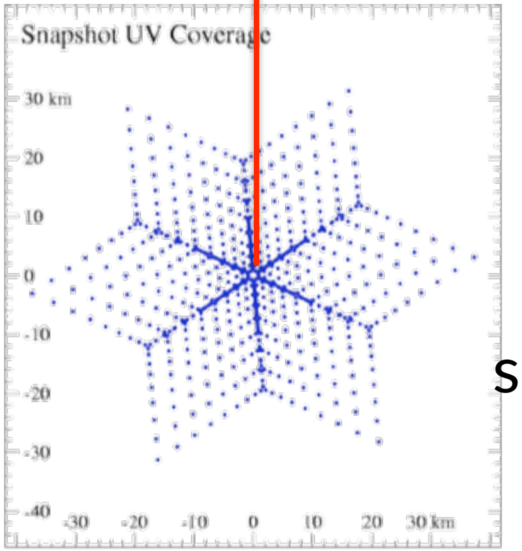

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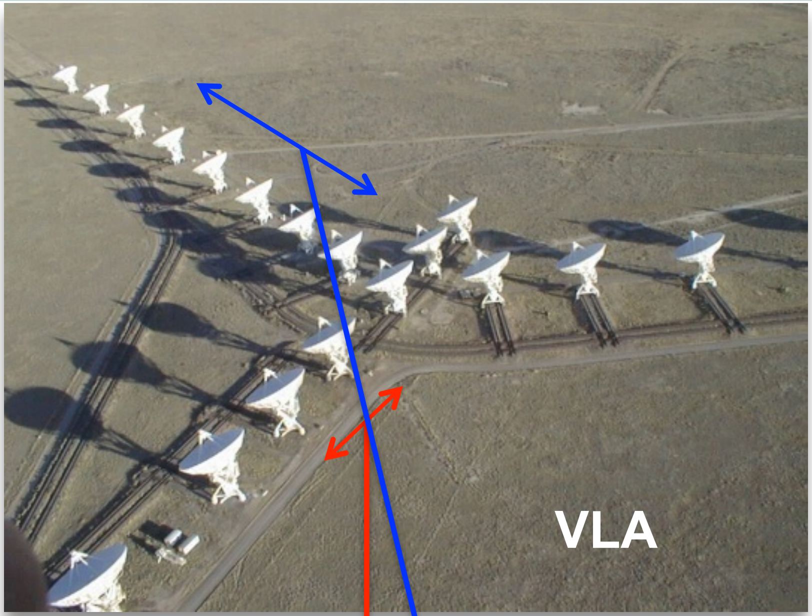

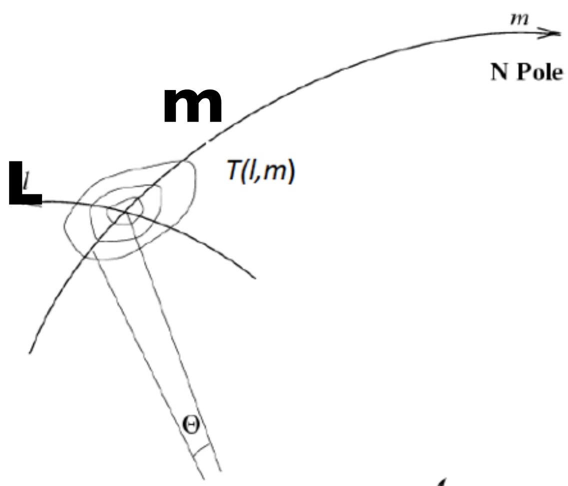

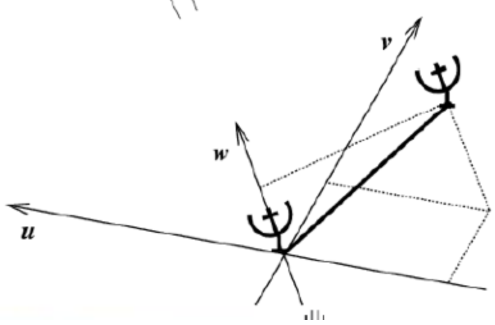

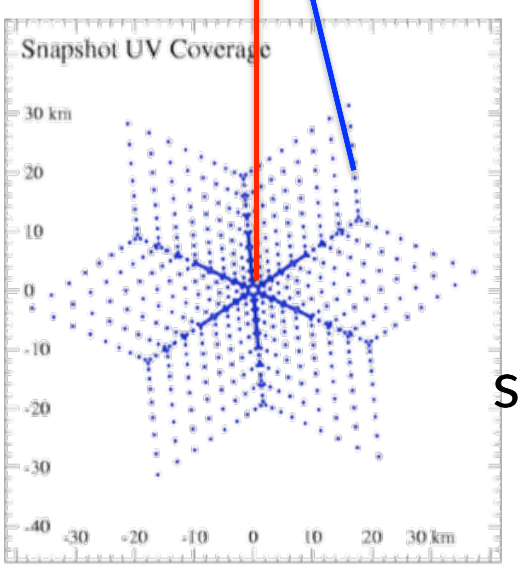

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Overview 4: Fourier link between the sky and the measurement

Along this course, we will establish an useful relationship linking the information of the sky, to a quantity we can measure:

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Visibility — *Sky intensity distribution* — *Fourier Kernel*

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$(u,v) \leftrightarrow (l,m)$ *Fourier pairs between Fourier space and image space*

$dldm$ *Section of the sky on which is computed the Fourier transform*

$ul + vm$ *Parameters of the 2D spatial filter associated with the Fourier kernel...*

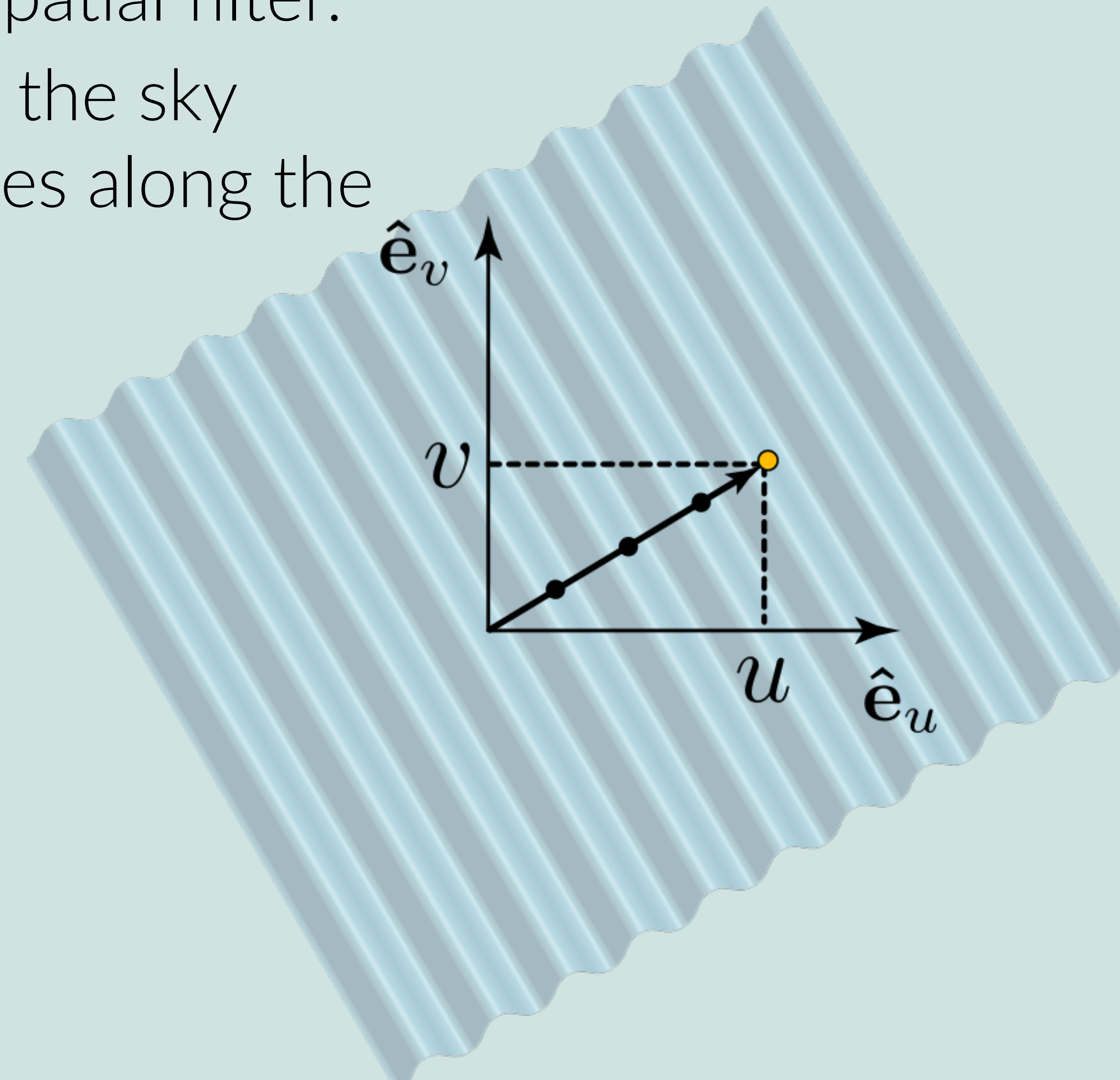
Overview 5: Measuring visibility as spatial filtering

$$\mathbf{f}_{u,v}^{l,m} = e^{-2j\pi(ul+vm)}$$

$$r_{uv} = \sqrt{u^2 + v^2}$$

The Fourier kernel acts as a spatial filter.

If (l,m) are the coordinates of the sky (u,v) are the spatial frequencies along the same axes.



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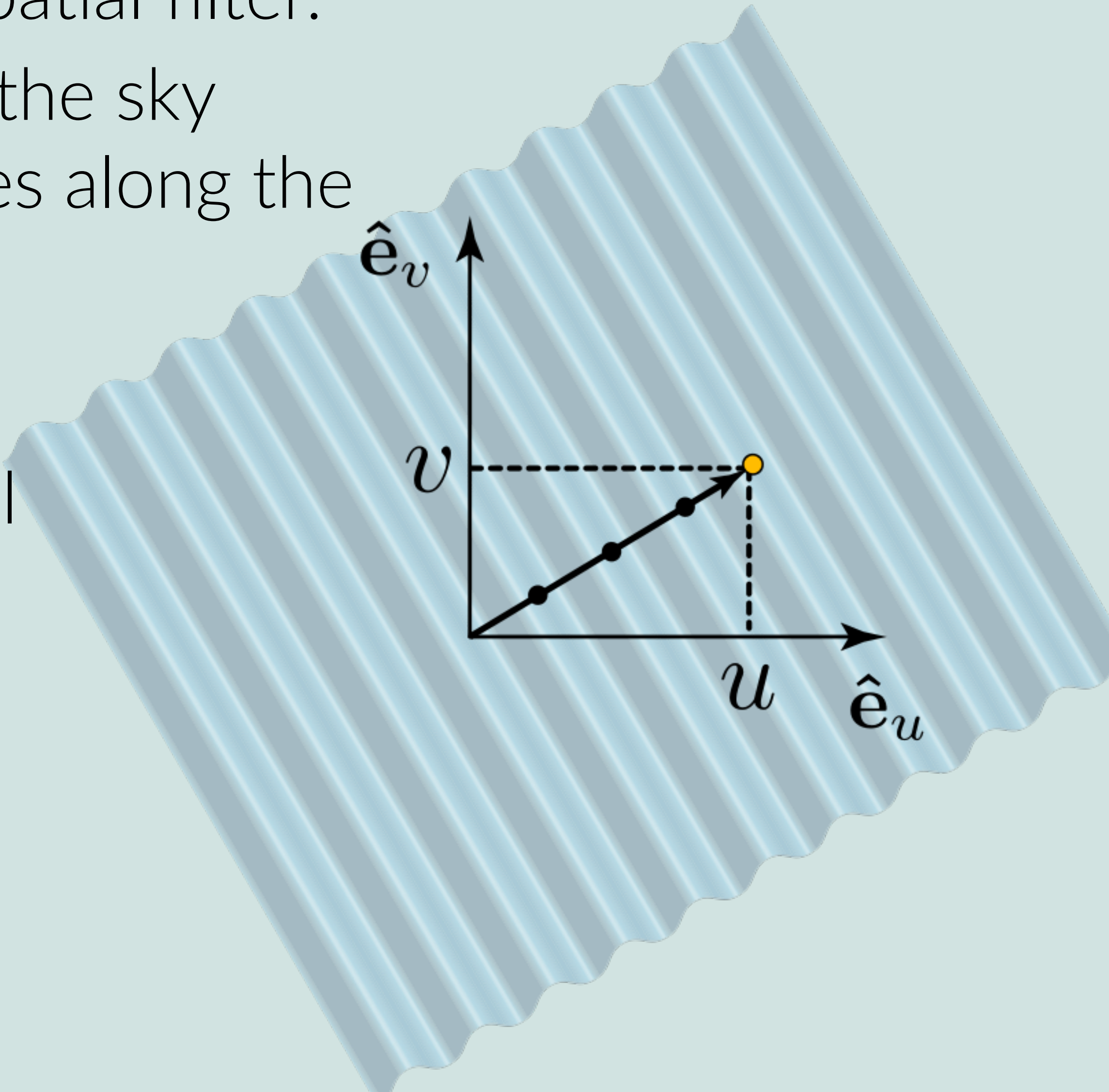

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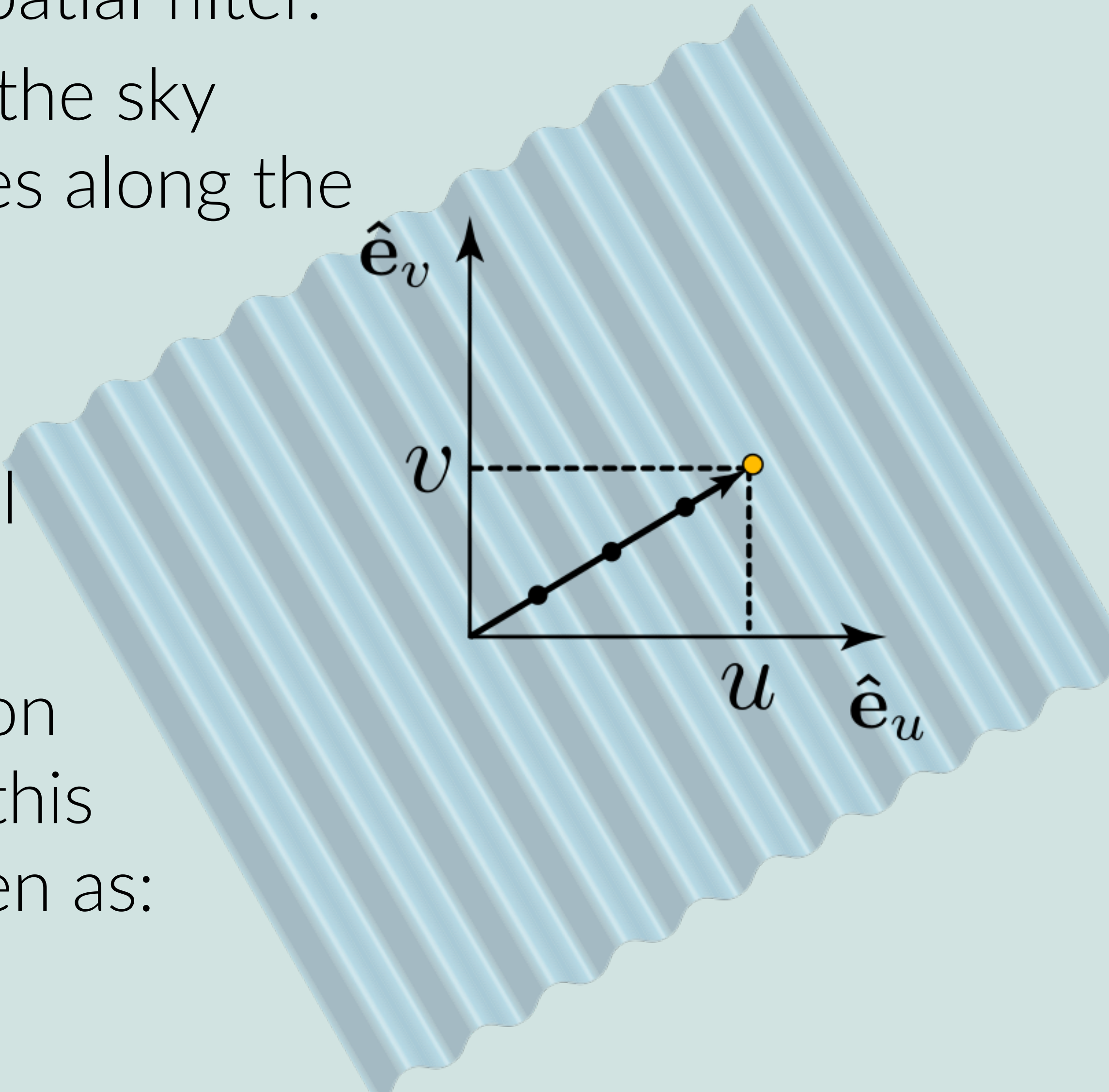

Fourier kernels are vectors in a basis with which a functional scalar product can be defined.

The visibilities are the collection of coefficients resulting from this scalar product. It can be written as:

$$\mathcal{V} = \langle \mathbf{I}_\nu \cdot \mathbf{f}_{u,v}^{l,m} \rangle$$

Outline of this chapter

Online lecture notes

We will focus this chapter mostly on the ***visibility space***

→ We will study the case of a simple **2-element interferometer** §4.2

→ We will link the interferometer measurement to the sky through the **visibility function** §4.3

→ We need means of representation of the **interferometric baseline** §4.1

→ We will link the visibility function and the sky through a **Fourier Transform relationship** (VC-Z theorem) §4.5

→ We will introduce how to **improve** the sampling of the visibility space with **time/frequency integration** §4.4

… to prepare the next chapter on the ***image space***

The 2-element interferometer

The 2-element interferometer : the “emitting” case

Let’s assume 2 point sources, *coherent* and *in phase* emitting 2 spherical waves

The interference zone is the location where the two waves can add *constructively*
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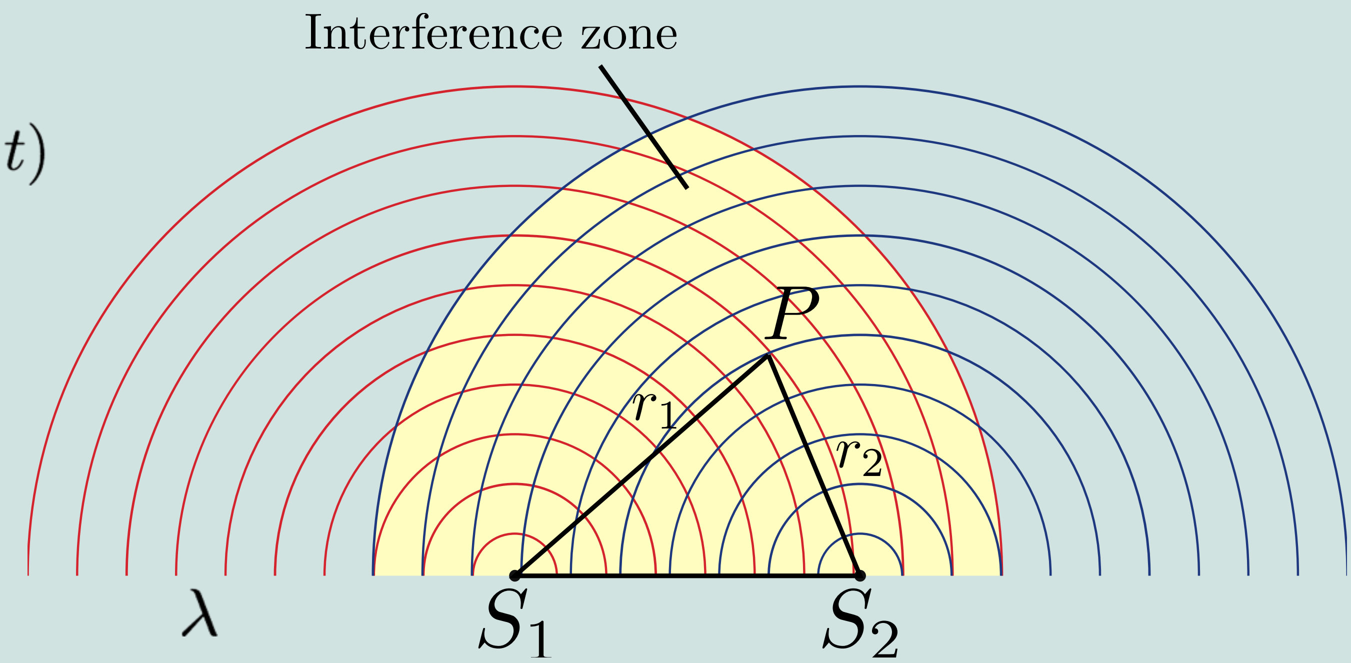

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Optical path difference

$$\Delta\Phi = 2\pi\frac{\Delta L}{\lambda}$$

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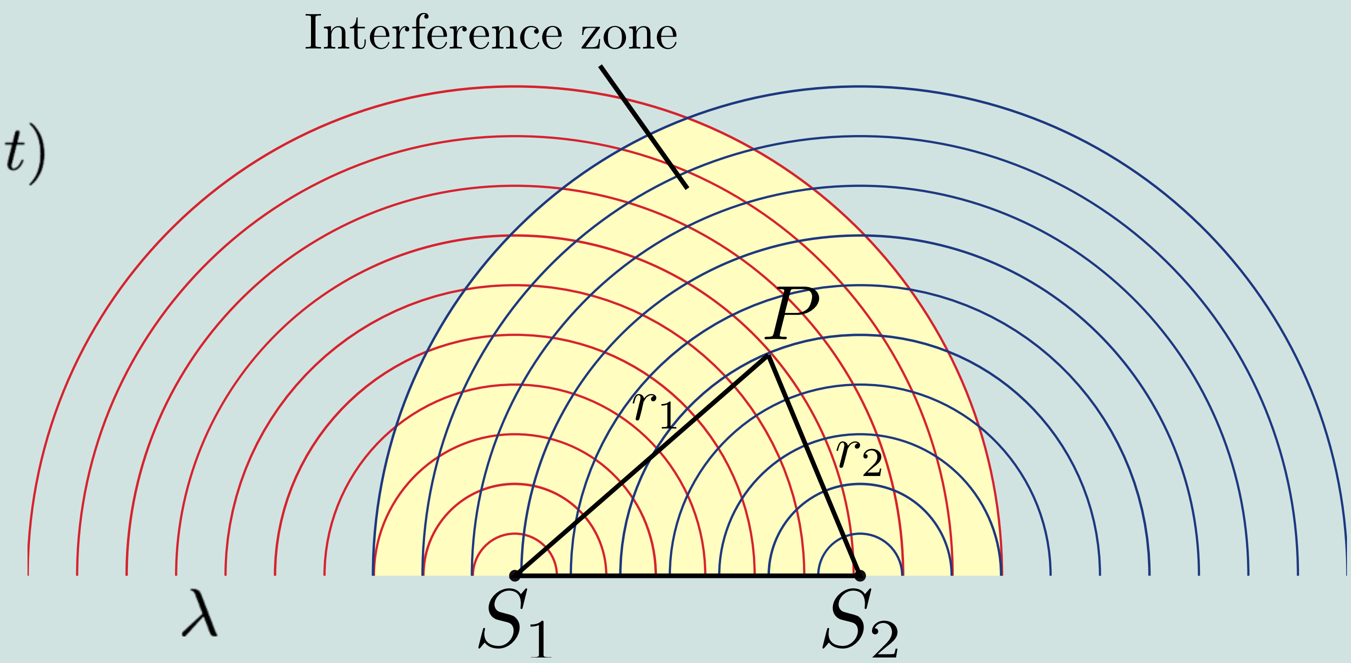

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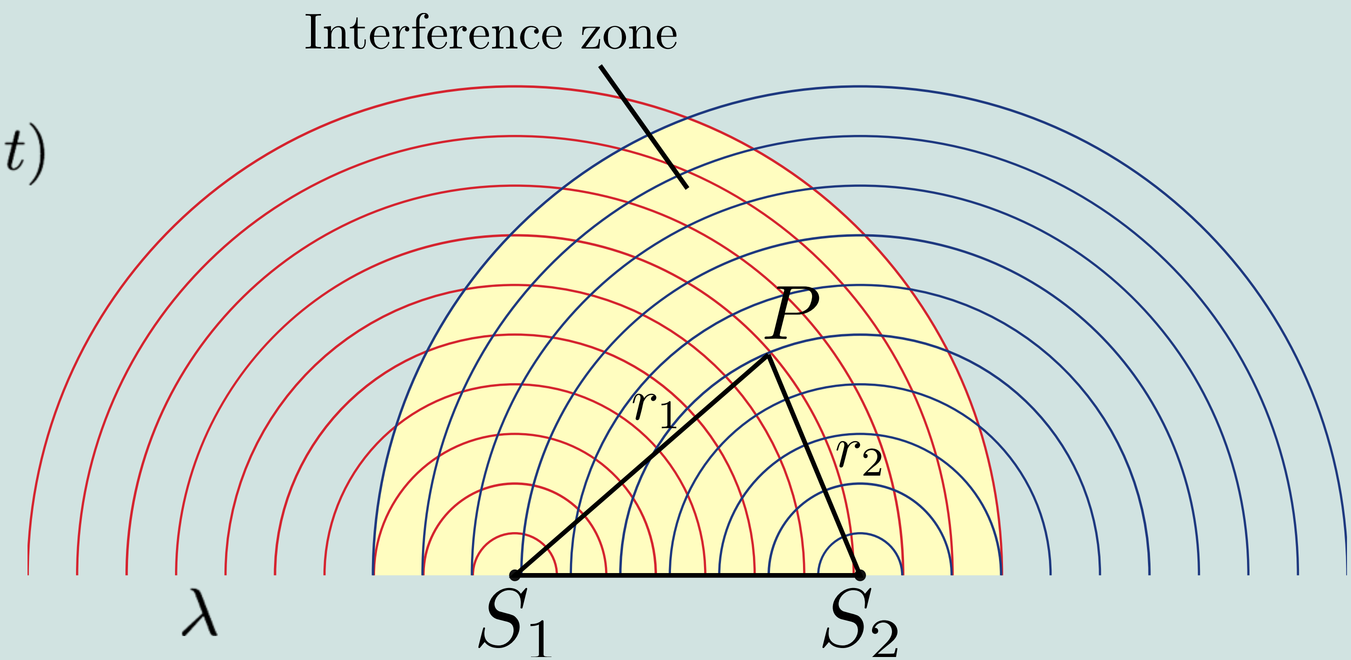

At point P, the total signal is the sum of the two signals coming from S_1 and S_2.

It depends on the relative phase delay of reception $\Delta\Phi$

$$s_P(t) = S_{0P}\cos(\omega t + \phi_{0P})$$

with $S_{0P} = \sqrt{S_{01}^2 + S_{02}^2 + 2S_{01}S_{02}\cos\Delta\Phi} = \sqrt{2S_0^2(1+\cos\Delta\Phi)}$

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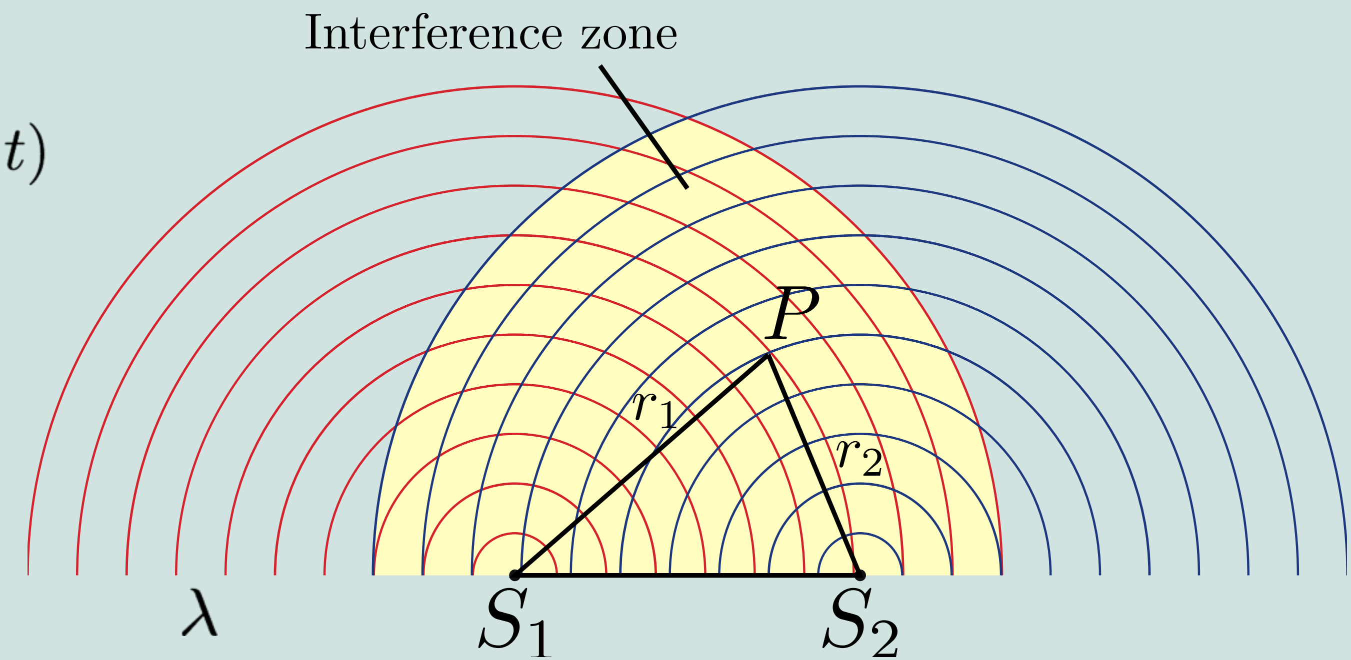

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interference pattern

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S_{0P} is maximal when $\cos\Delta\Phi = +1$ i.e. $\Delta\Phi = 2m\pi$ $m \in \mathbb{Z}$

$\Delta L = m\lambda$

➡ "Bright fringes"

constructive interference

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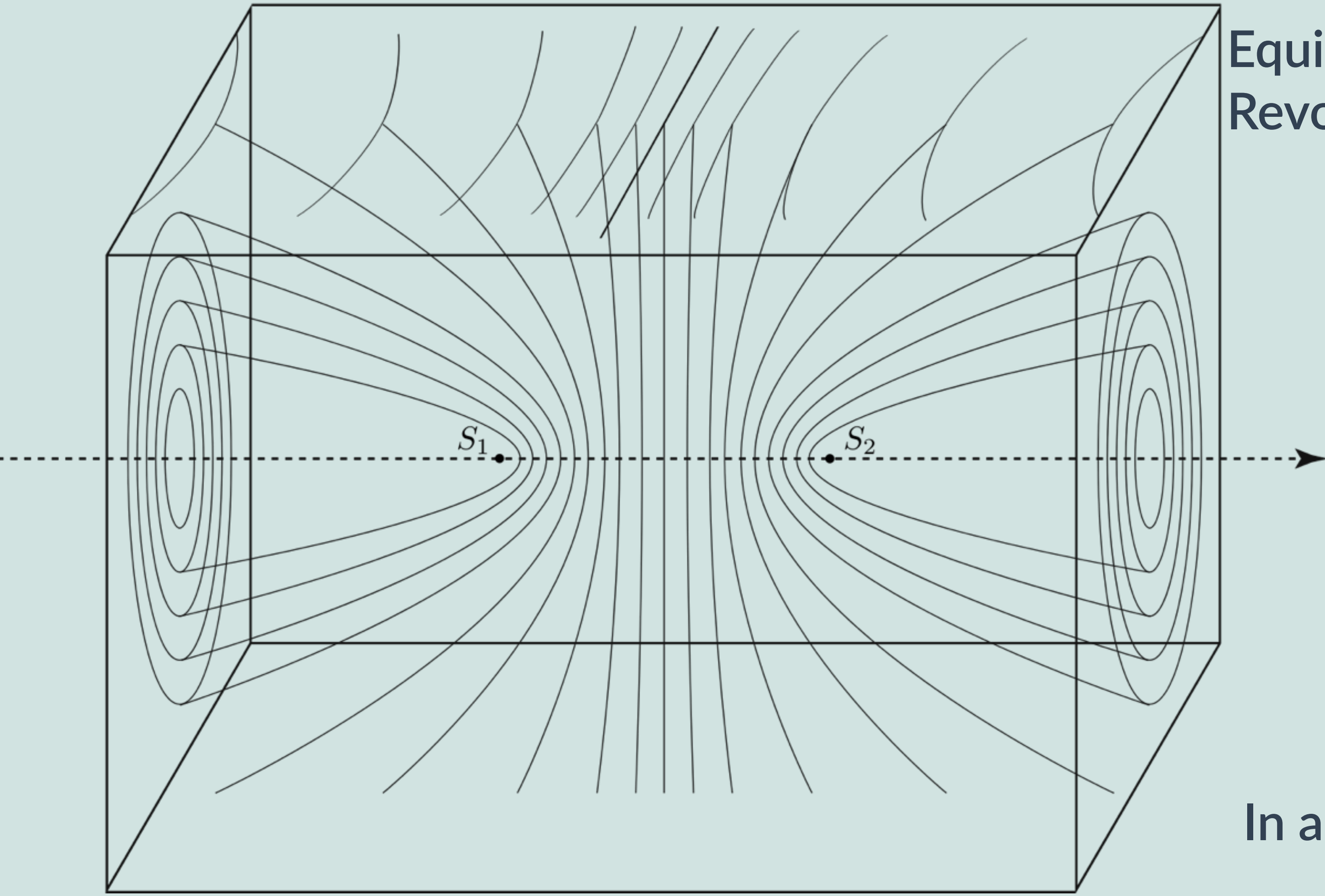
In space, a fringe is defined as the location of points where the phase $\Delta\Phi$ is constant, or $S_2P - S_1P = \text{const}$

The 2-element interferometer : the “emitting” case

In space, the interference pattern can be collected on a screen

In a plane // to S_1S_2

Linear fringes



Equiphase surfaces =
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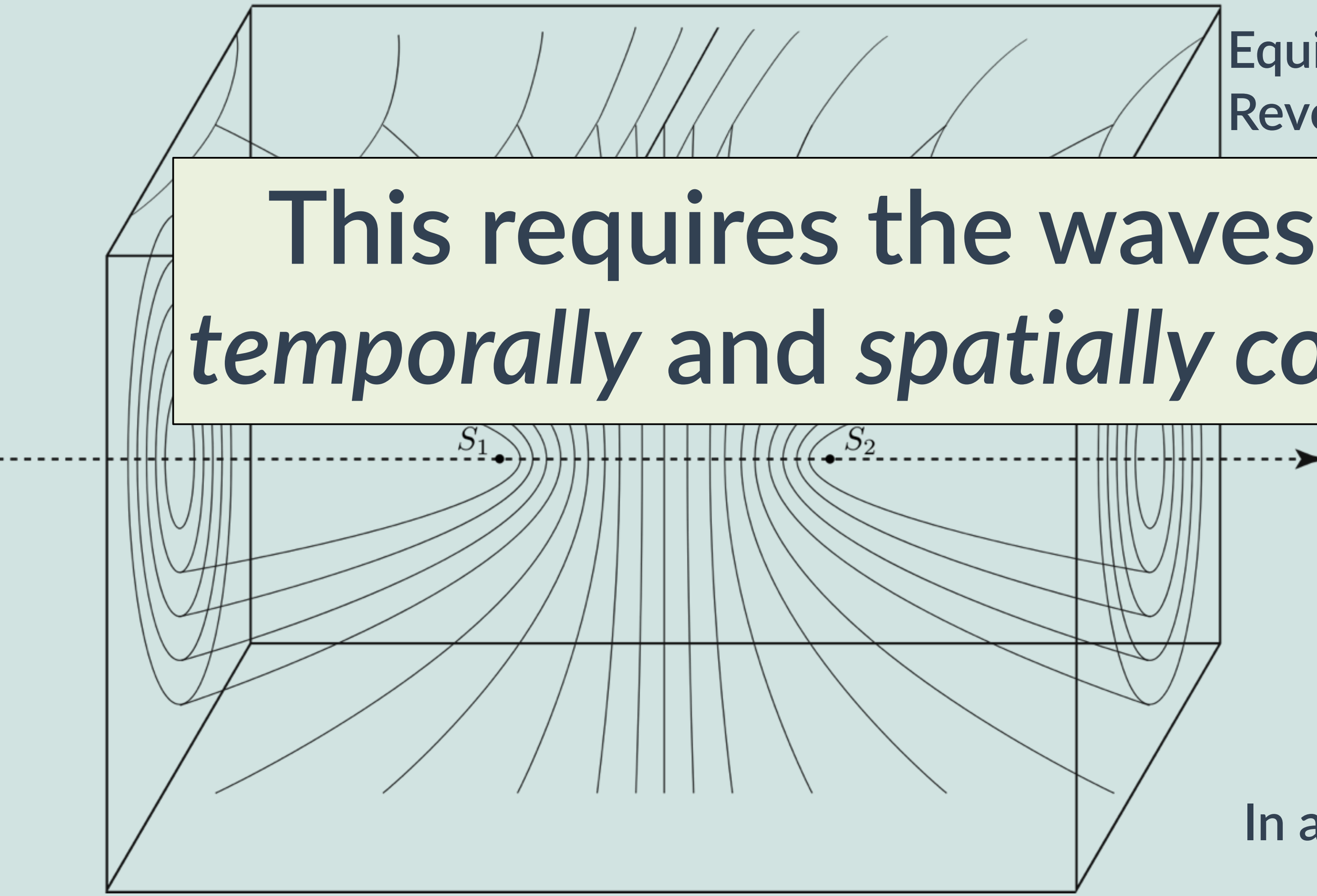

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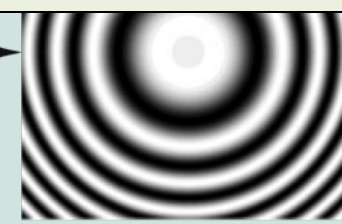
This requires the waves to be *temporally* and *spatially coherent !*



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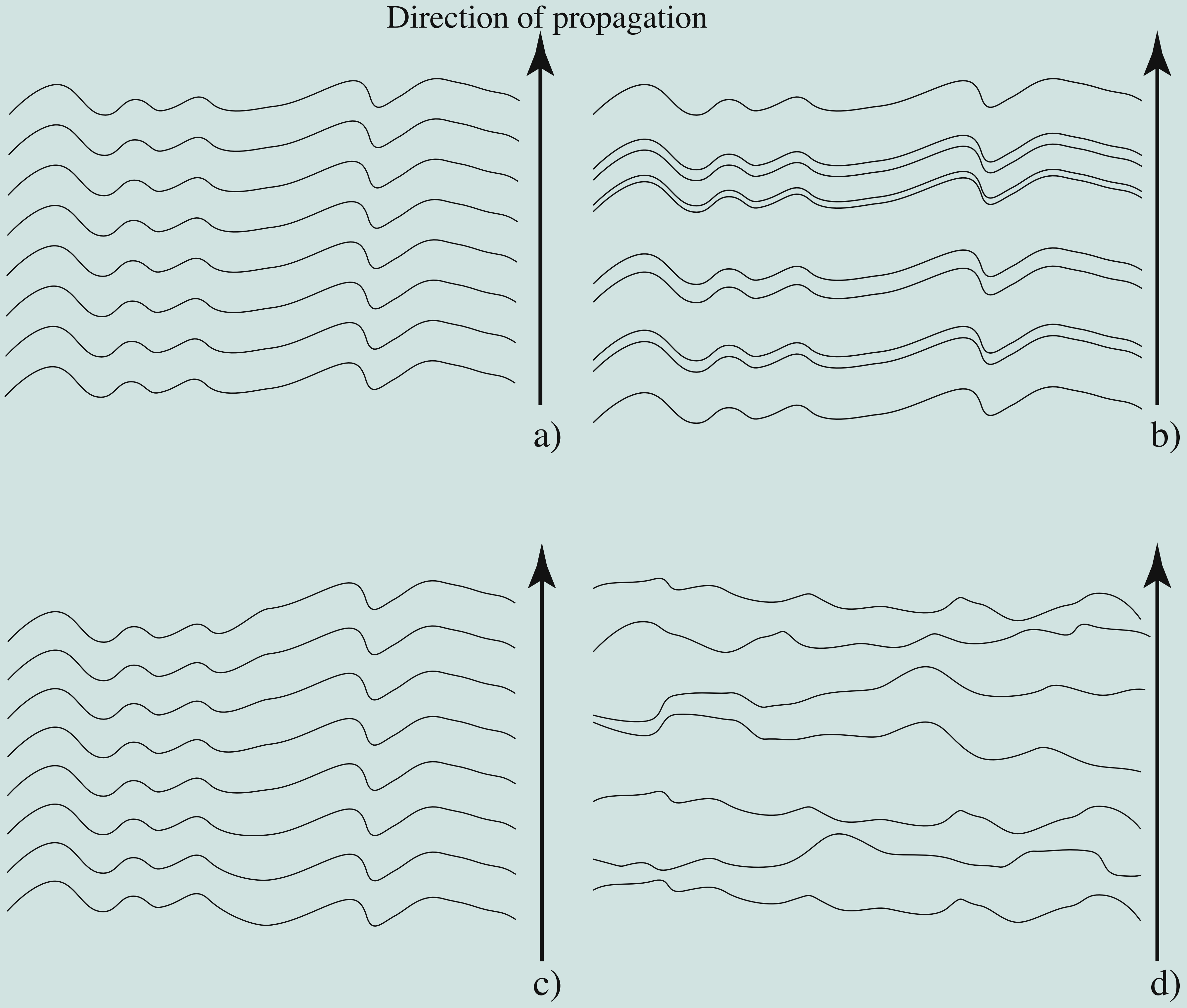
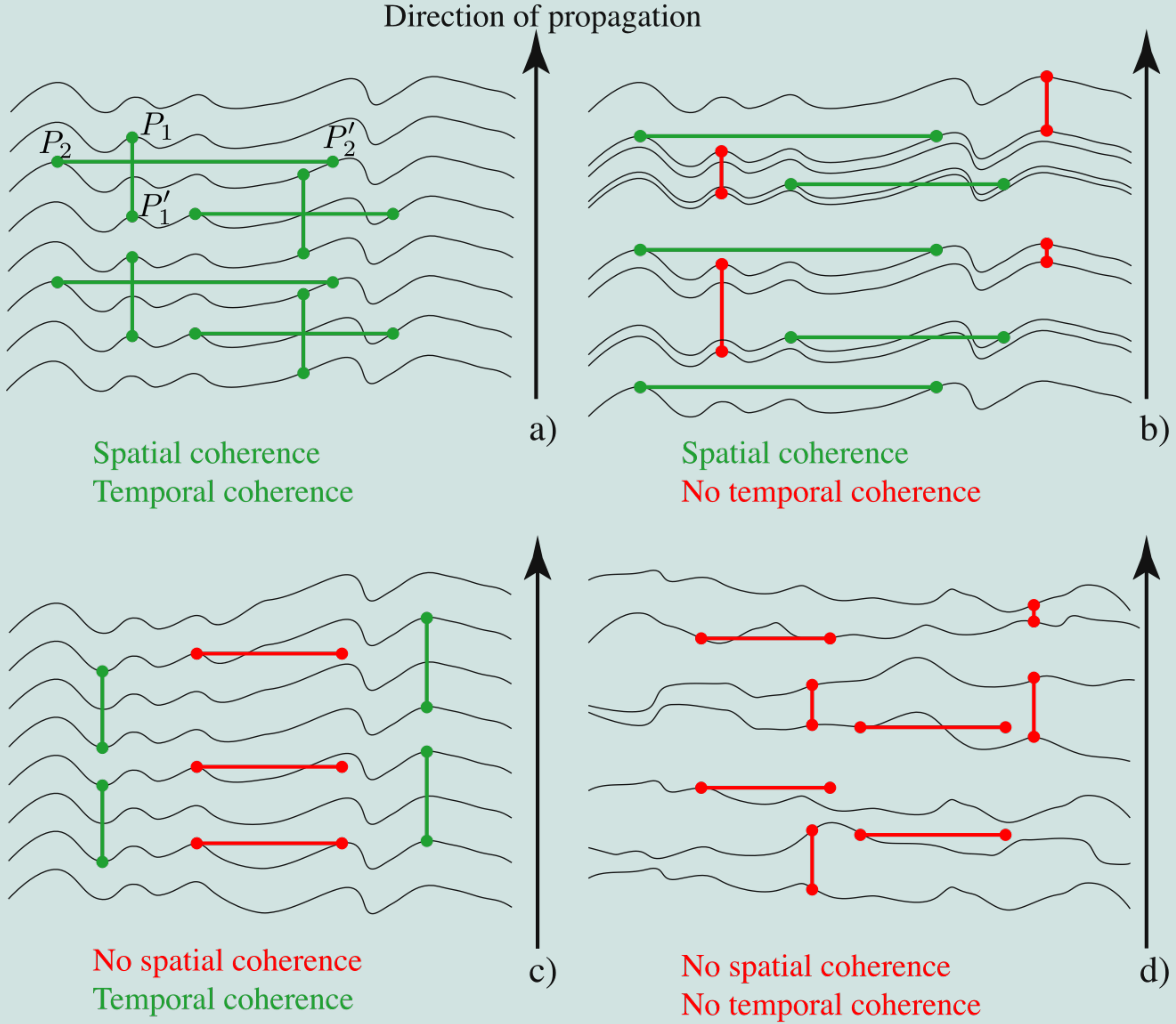

Direction of propagation
a)
b)
c)
d)


Direction of propagation
P_1
P_2
P_2'
P_1'
a)
Spatial coherence
Temporal coherence
b)
Spatial coherence
No temporal coherence
c)
No spatial coherence
Temporal coherence
d)
No spatial coherence
No temporal coherence

And now, in radio
In the « receiving » case

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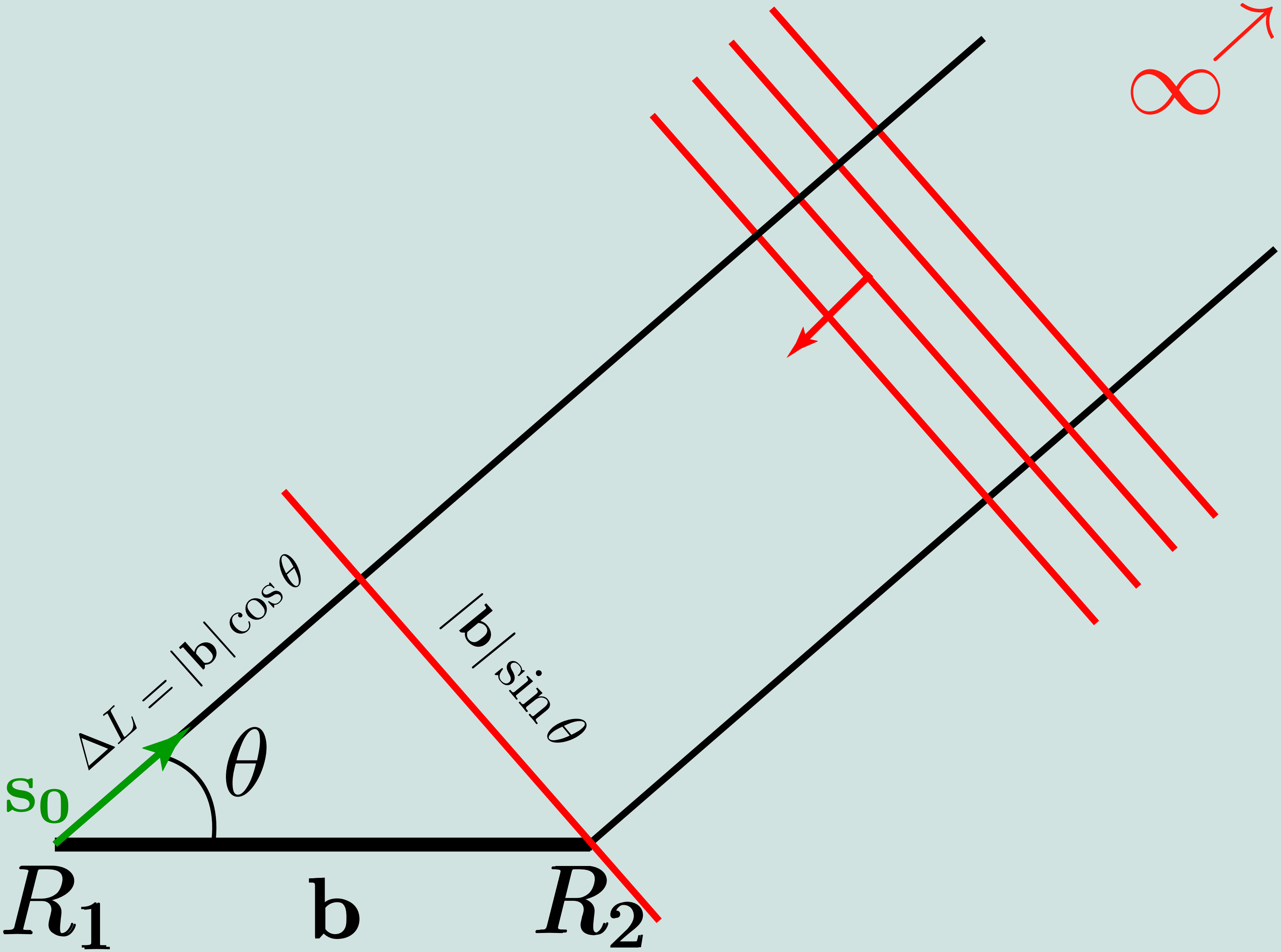

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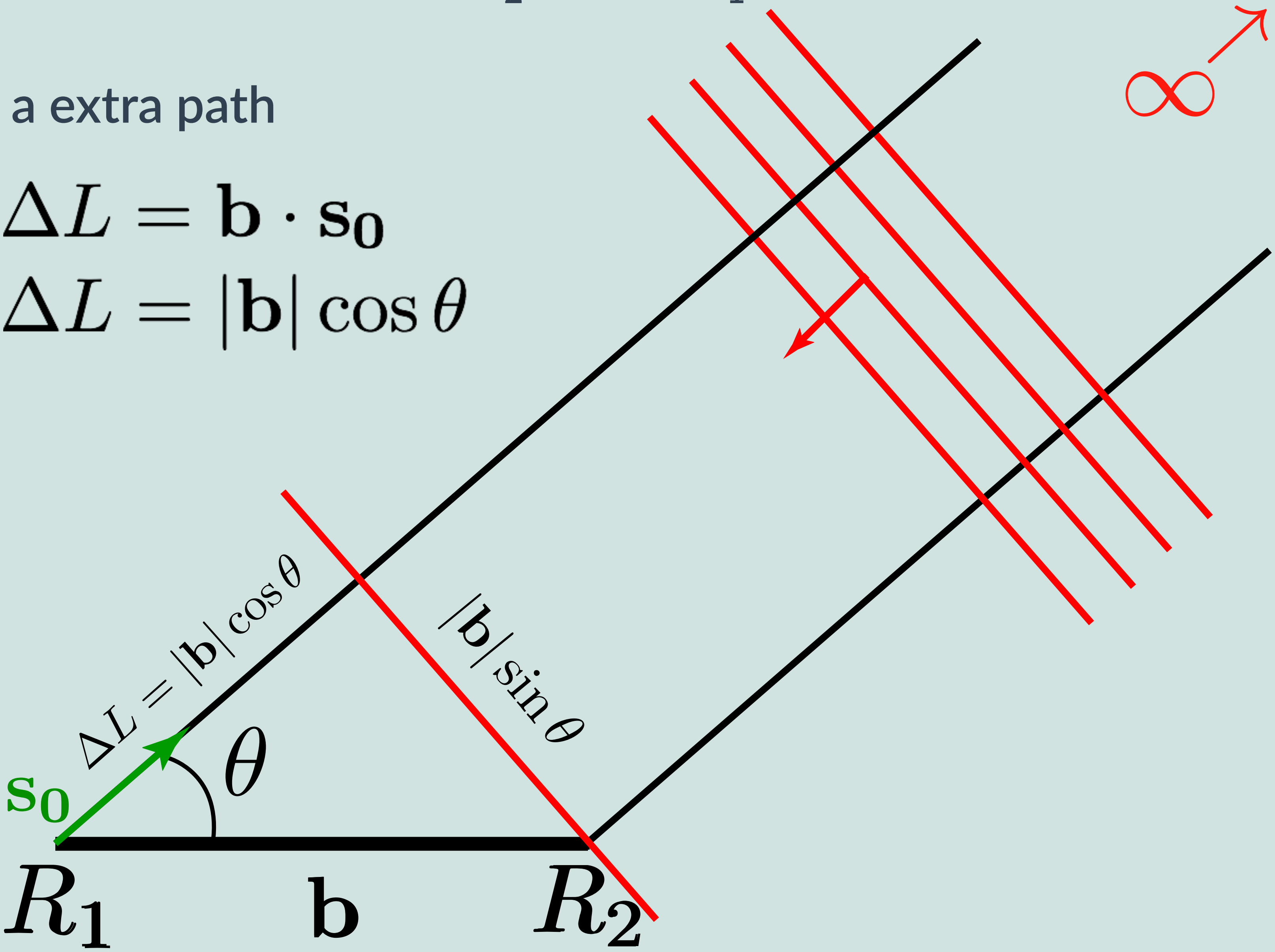
The signal will reach R_2 before R_1 and creates:

a extra path

$$\Delta L = \mathbf{b} \cdot \mathbf{s_0}$$

$$\Delta L = |\mathbf{b}| \cos\theta$$



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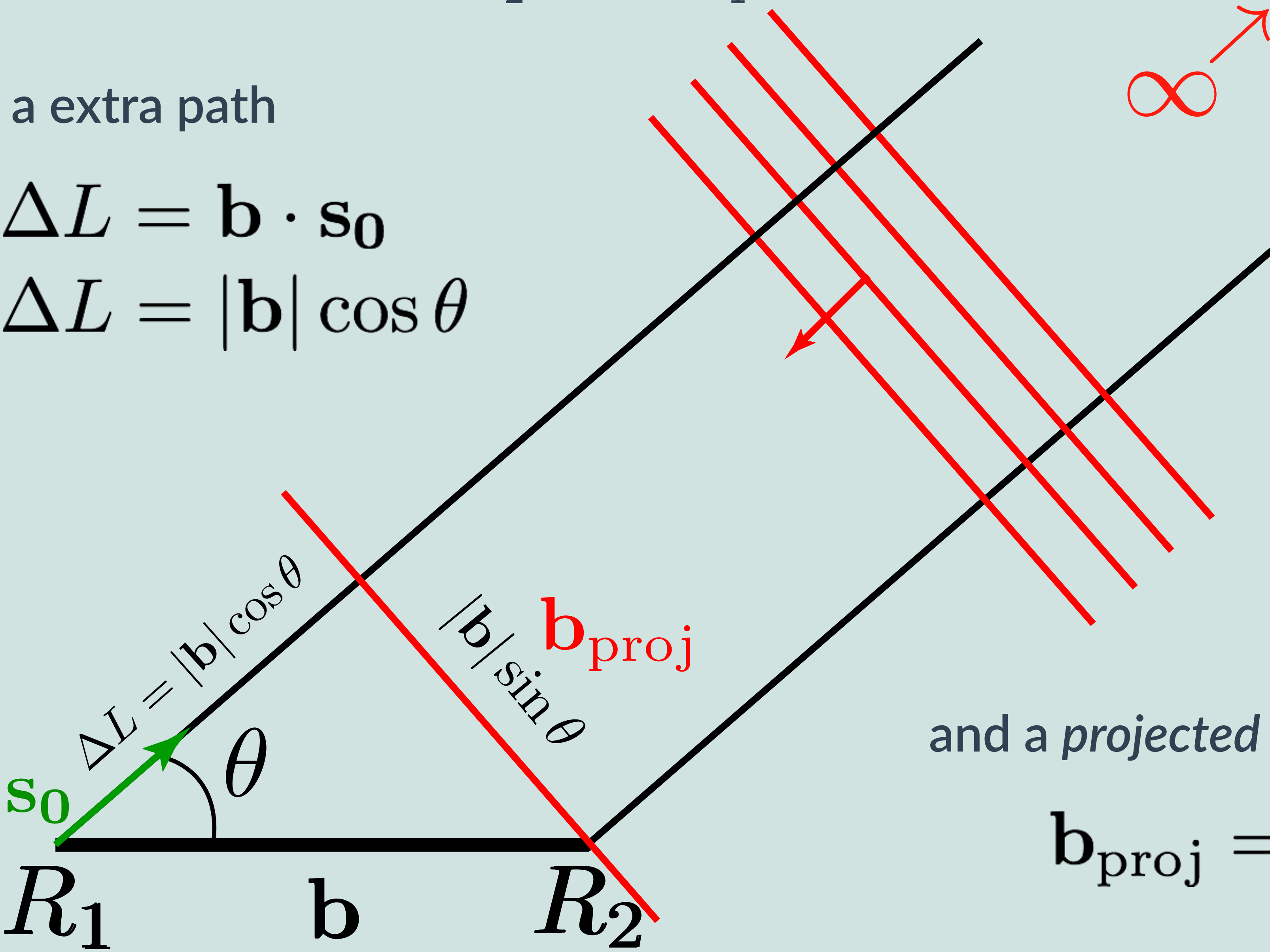

and a *projected* baseline

$$\mathbf{b}_{\mathrm{proj}} = |\mathbf{b}| \sin\theta$$

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Let V_1 and V_2 the measured voltages at R_1 and R_2

$$V_1 = V_{01}\cos(\omega t + \varphi_1) \quad V_2 = V_{02}\cos(\omega t + \varphi_2)$$

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R_1 as the reference receiver, shifting the origin of time so that $\varphi_1 = 0$

We can rewrite the expression of the voltages as

$$V_1 = V_{01}\cos(\omega t), \quad V_2 = V_{02}\cos(\omega t + \varphi_2 - \varphi_1)$$

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$\varphi_2 - \varphi_1$ is linked to ΔL the extra path required for the plane

wave to go from R_2 to R_1.

The 2-element interferometer : the “receiving” case

Let V_1 and V_2 the measured voltages at R_1 and R_2

$$V_1 = V_{01}\cos(\omega t + \varphi_1) \quad V_2 = V_{02}\cos(\omega t + \varphi_2)$$

We use $\mathbf{s_0}$ as a reference direction of the incoming wave

R_1 as the reference receiver, shifting the origin of time so that $\varphi_1 = 0$

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$$\varphi_2 - \varphi_1 = \Delta\Phi = \frac{\omega}{c}\Delta L \quad \text{with} \quad \Delta L = \mathbf{b}\cdot\mathbf{s_0}$$

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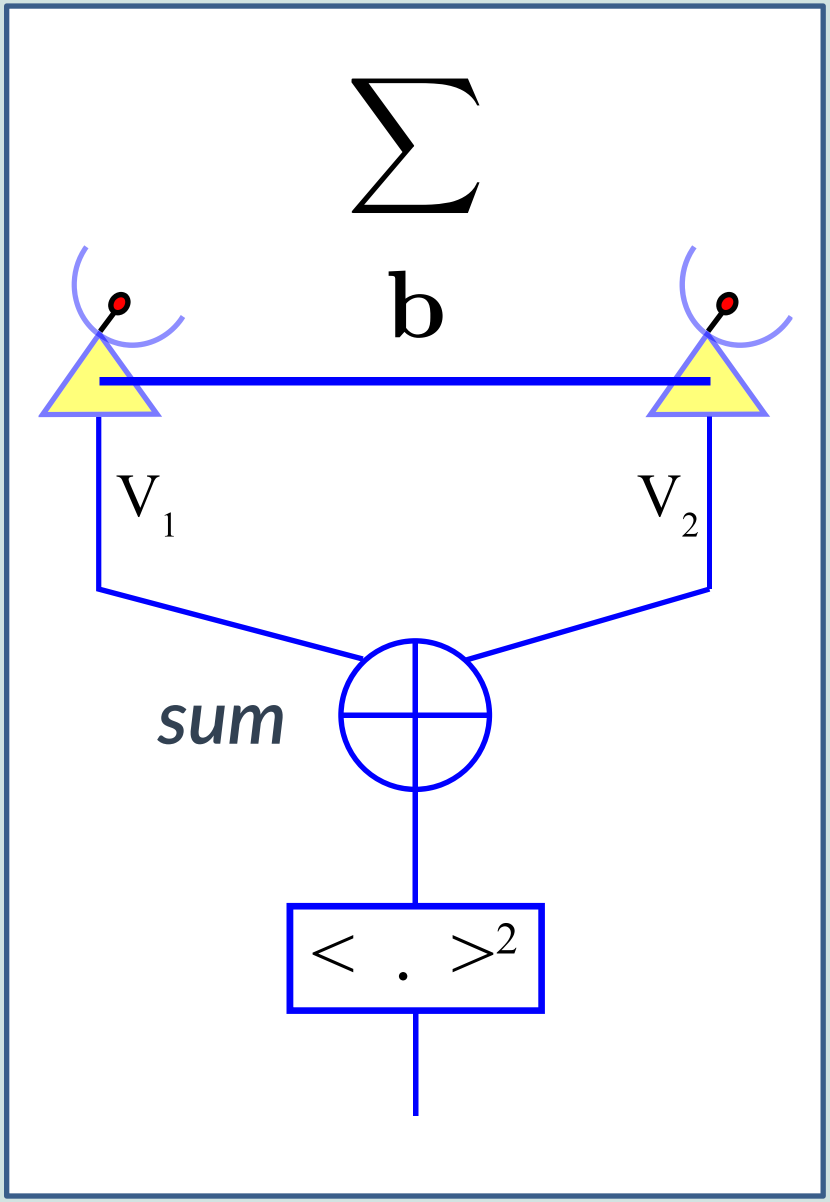
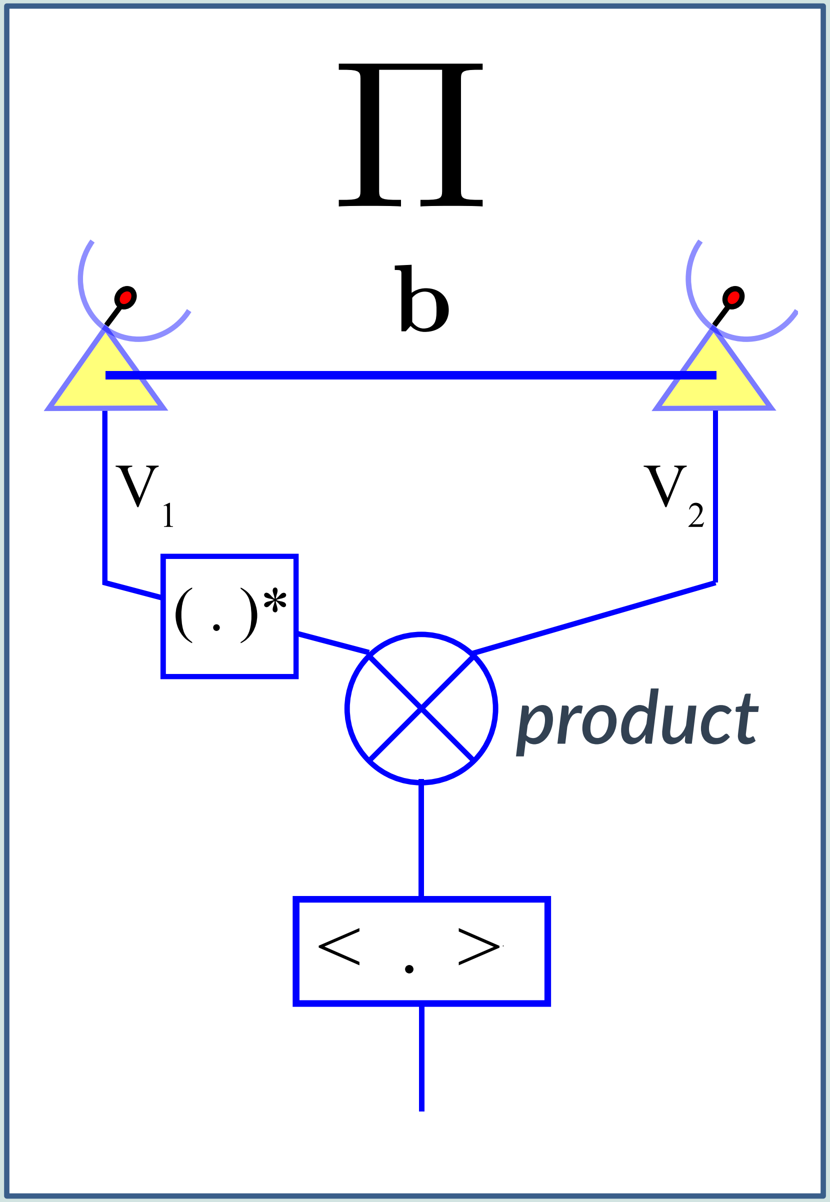
Now that we defined the expression of the voltages, let’s see how we can combine them to generate the response of an interferometer.

The $\sum$ & $\prod$ interferometers

The 2-element interferometer : The $\sum$ & $\prod$ interferometers

There are mainly two ways to combine the measured voltages





The 2-element interferometer : The $\sum$ interferometer

The sum of the two voltages leads to a similar relationship as in optical interferometry.

$$A = \sqrt{(V_1 + V_2)^2} = \ \dots \ = \sqrt{2V_0^2(1 + \cos\Delta\Phi)} \text{ with } \Delta\Phi = \varphi_2 - \varphi_1$$

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It represents the average level of the voltage product between R_1 and R_2 and therefore depends on their properties (e.g. electrical, effective geometry)

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$$C = V_0^2 \frac{\langle \cos(2\omega t + \tau) + \cos(\omega \tau) \rangle_t}{2}$$

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It is equivalent to filter the signals with a low-pass filter which role is to remove its fast-varying component.
We call the remaining quantity the correlation given by a cosine correlator.

$$C_{\cos} = \frac{V_0^2}{2} \cos \omega\tau$$

The 2-element interferometer : The Π interferometer

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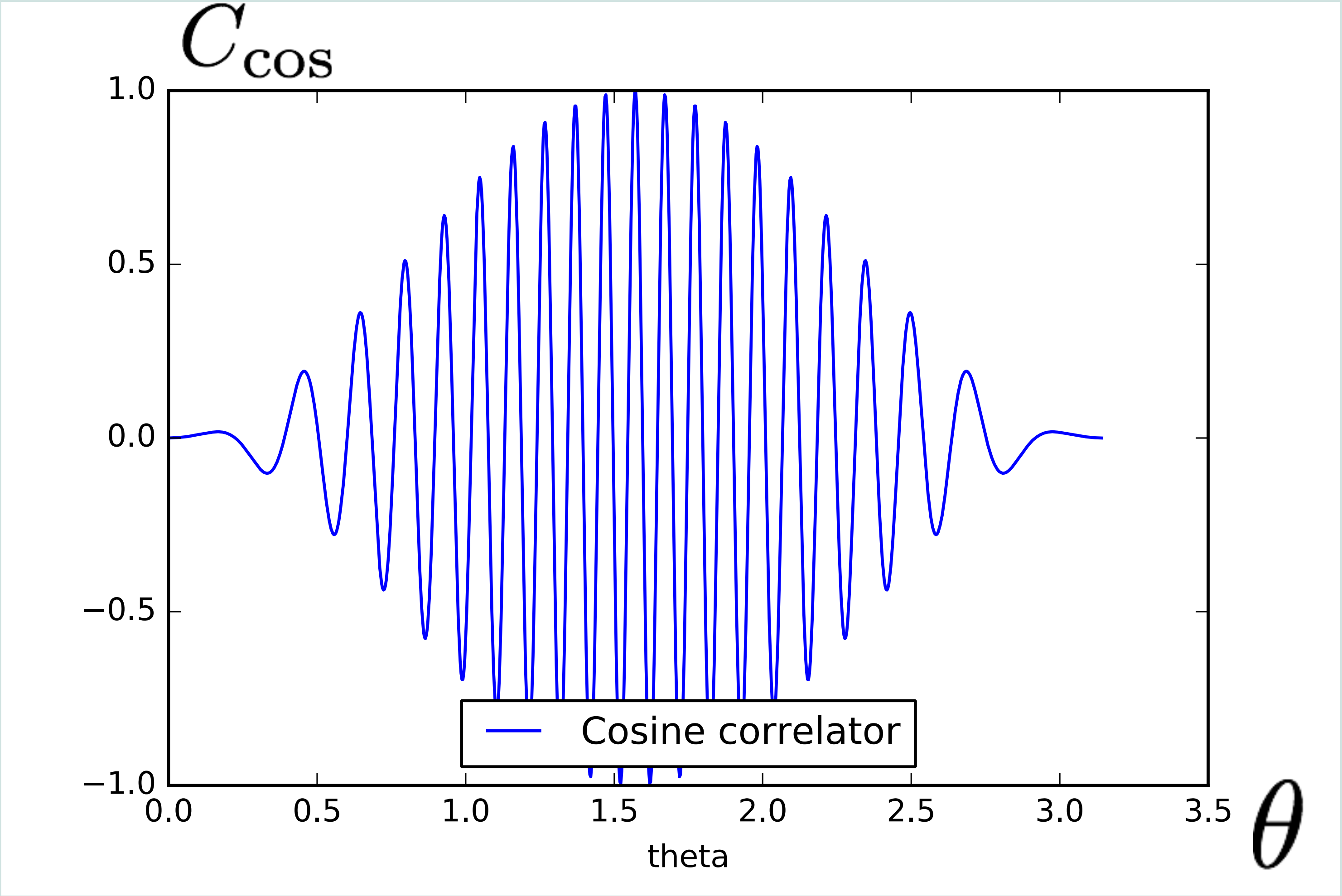
This correlation depends on the delay τ

It describes a fringe pattern in the sky

Which can be modulated for example by the antenna pattern



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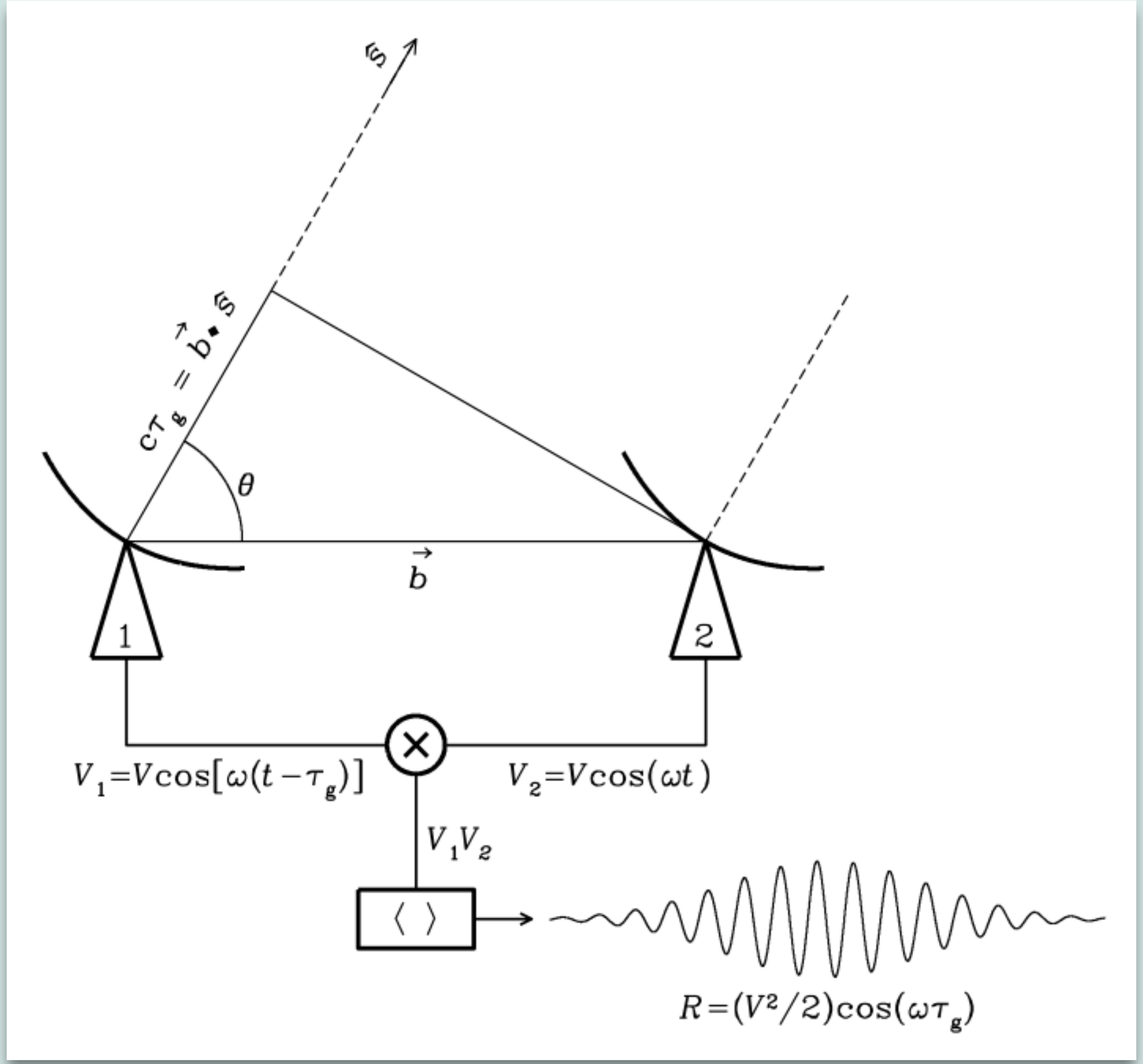


Π interferometer

&

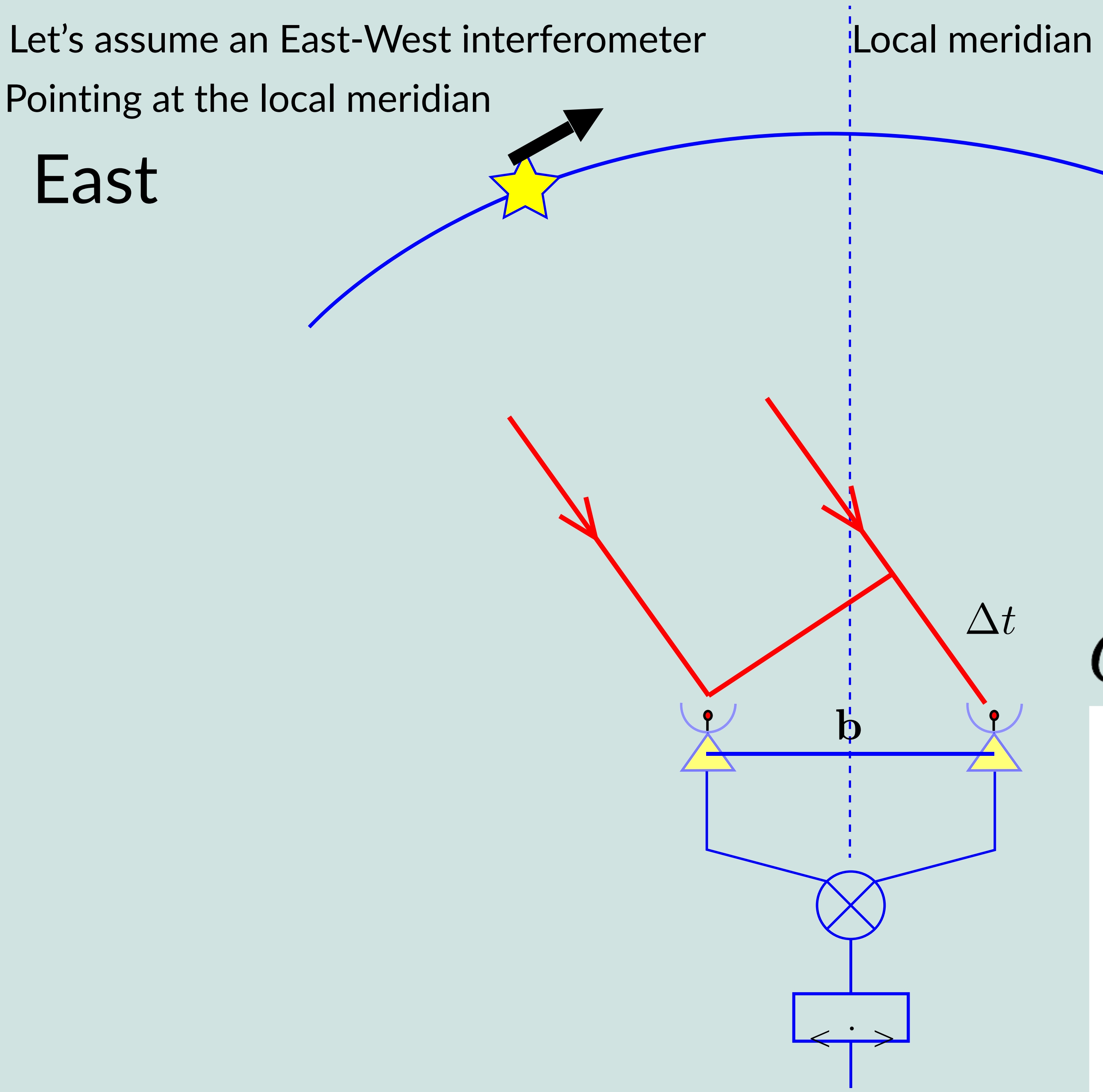
« Untracked »

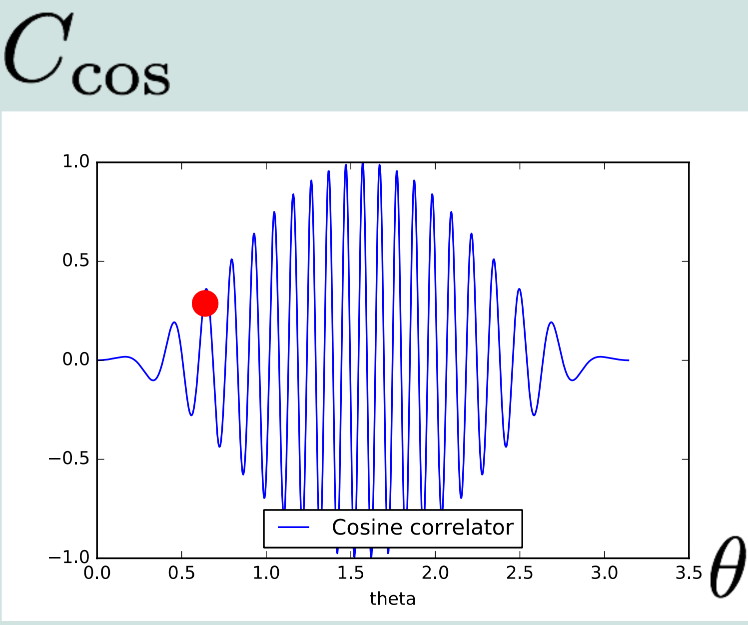
source

The 2-element interferometer : Untracked source

Let's assume an East-West interferometer

Pointing at the local meridian





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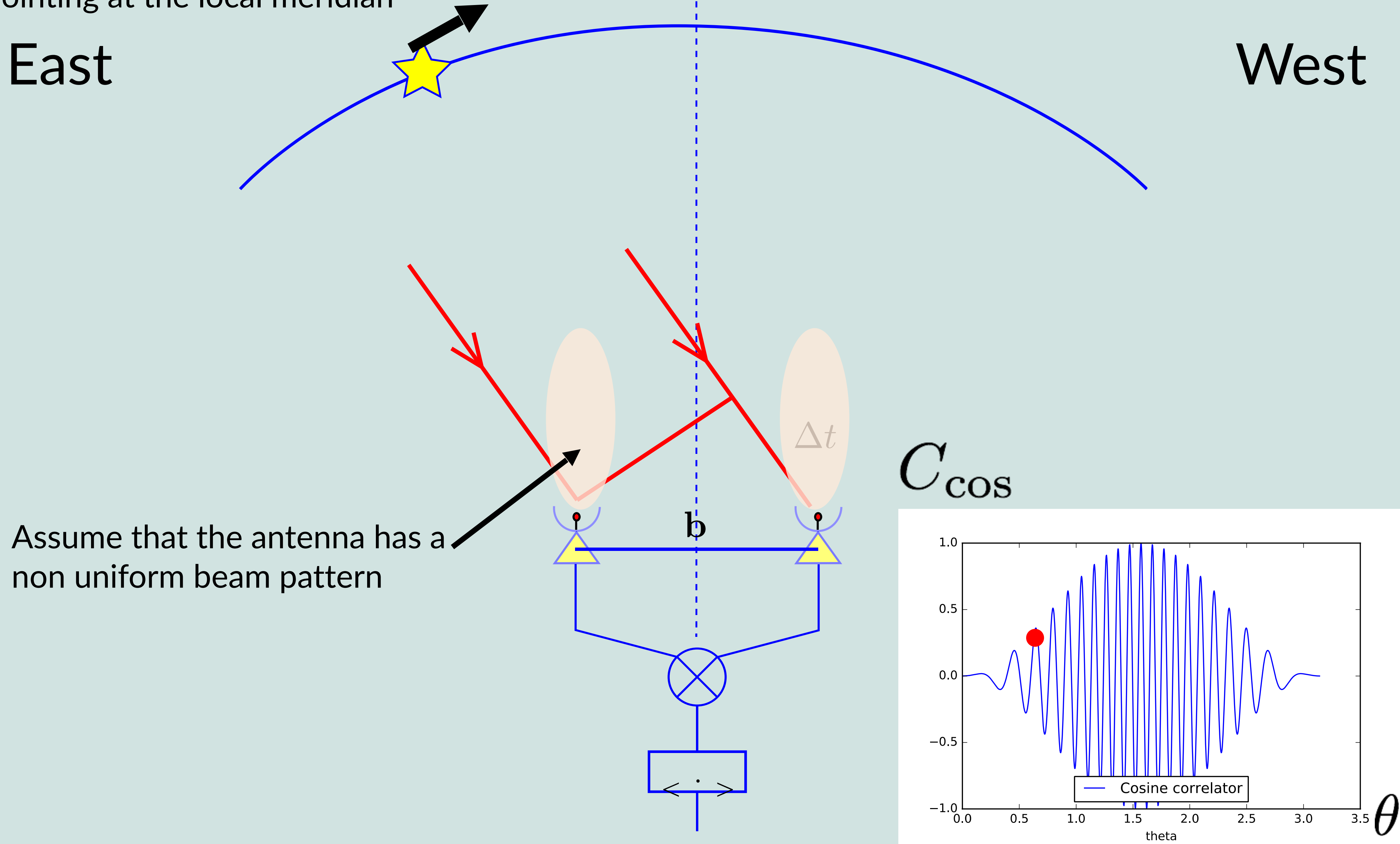

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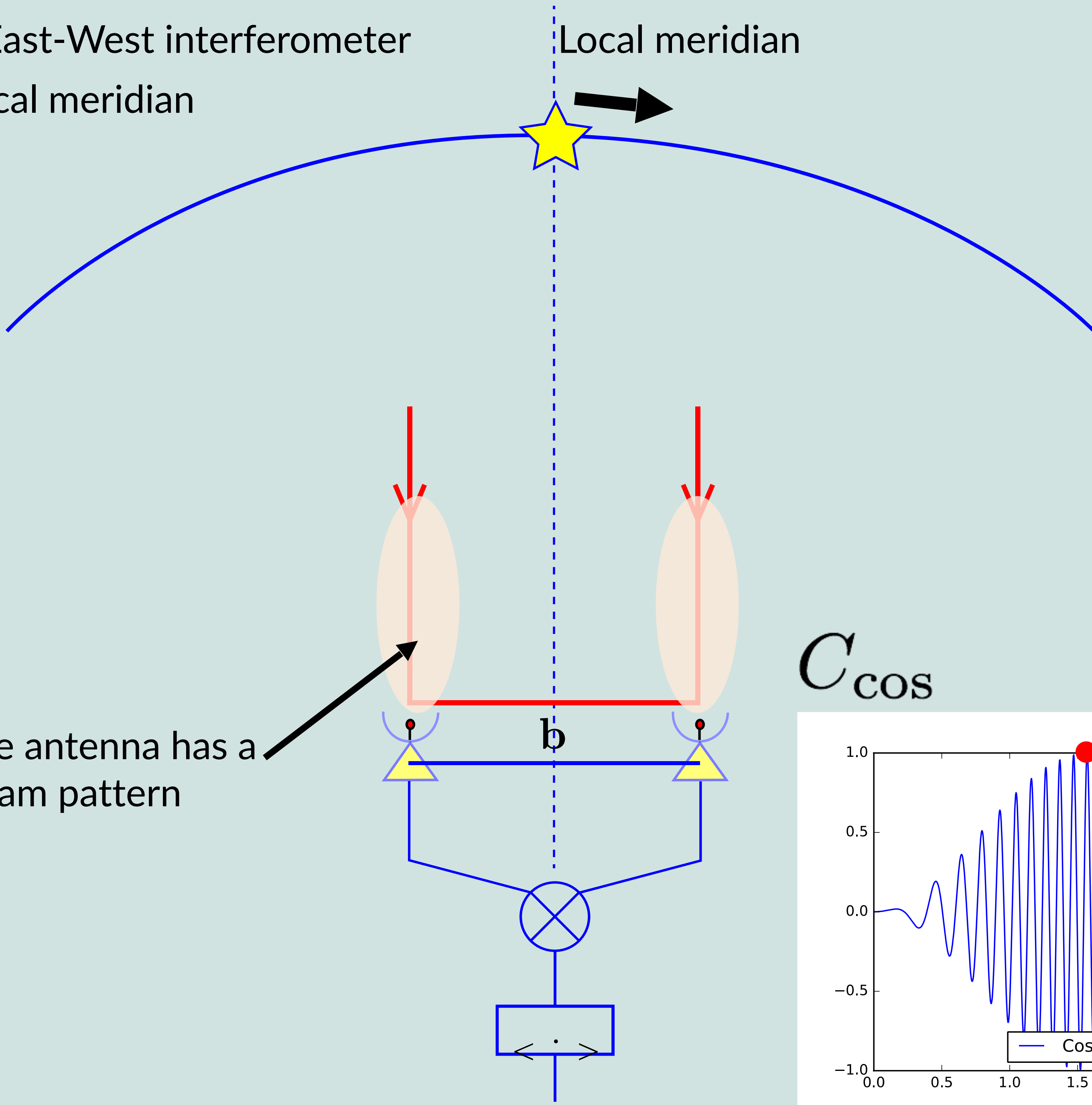
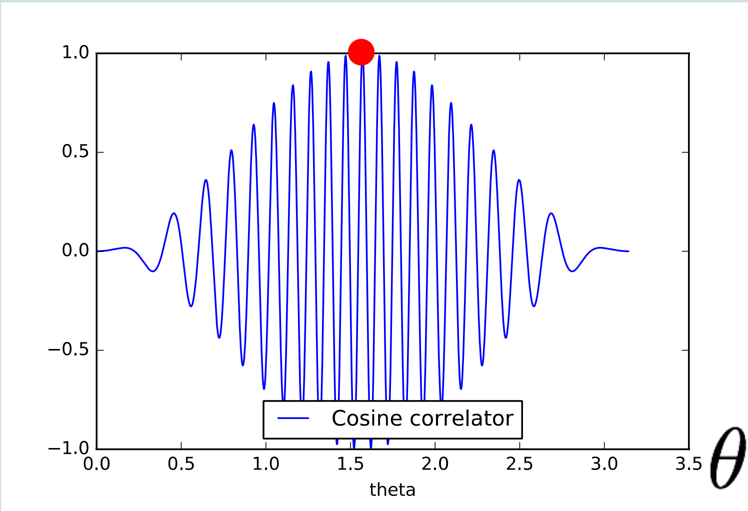


$C_{\cos}$



θ

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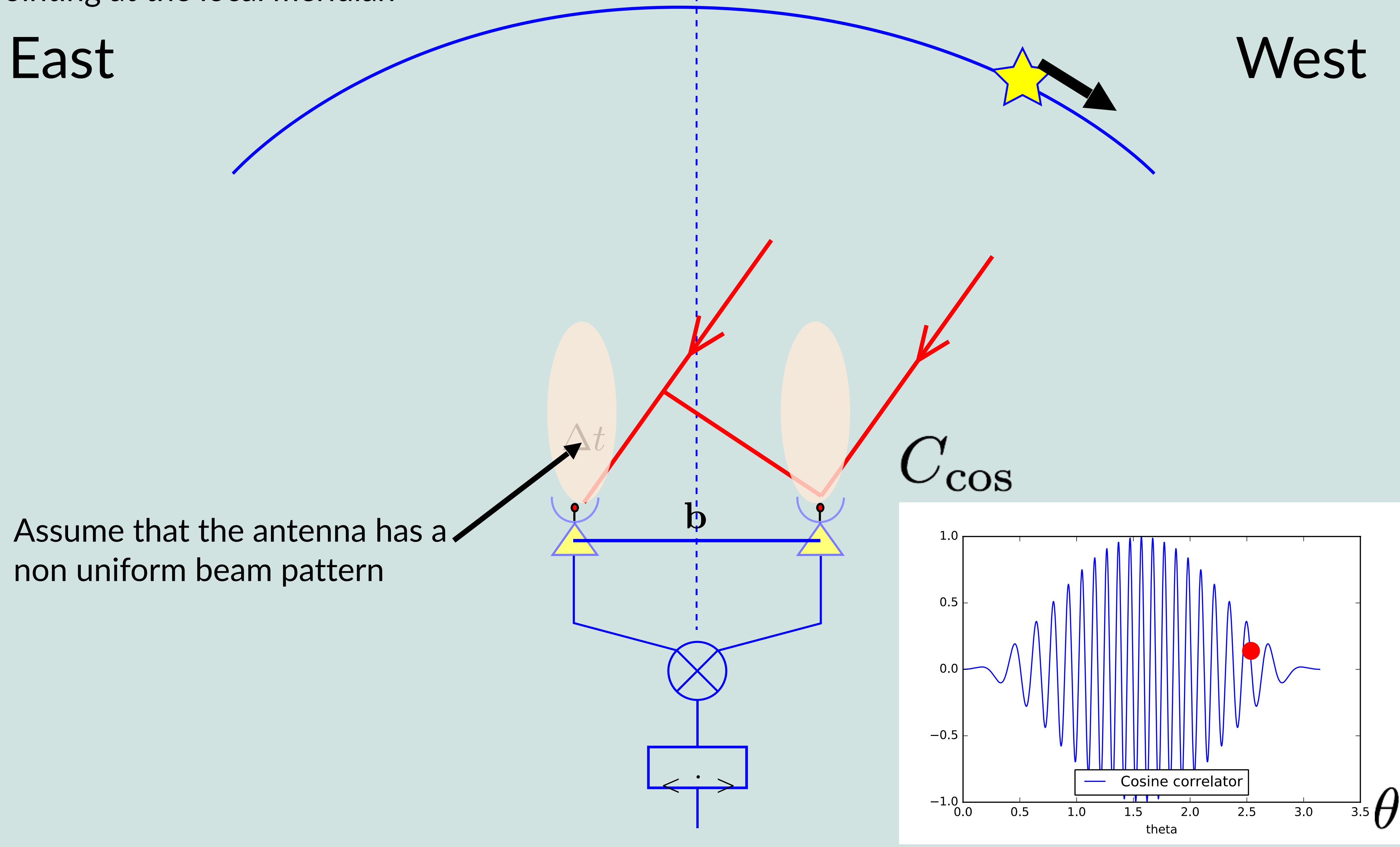

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And its derivative, the fringe rate:

The fringe rate $$|\frac{d\phi}{d\theta}| = \frac{2\pi}{\lambda}|\mathbf{b}\sin\theta| = \frac{2\pi}{T_f}$$

The fringe period T_f

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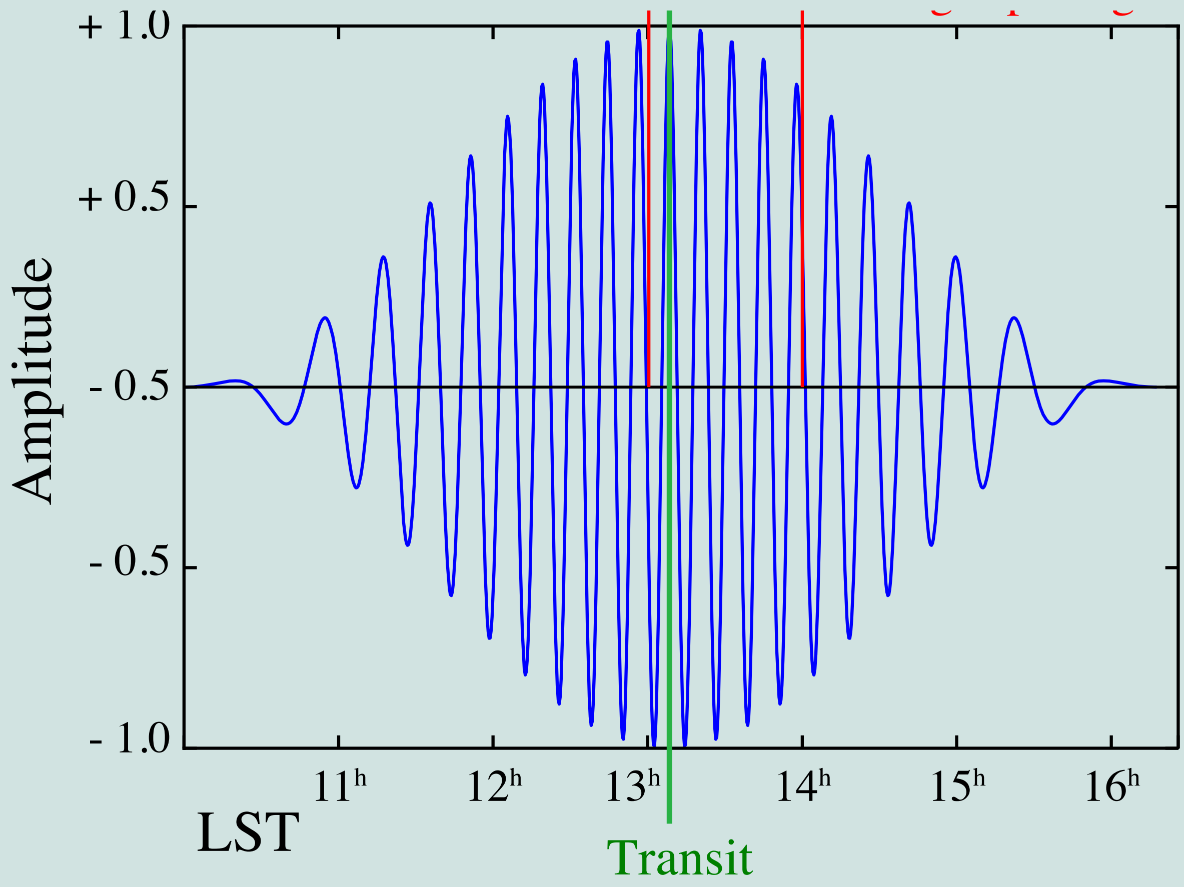
As a consequence, the correlation is sensitive to spatial variations of spatial period T_f.

which means that a 2-element interferometer acts as spatial filter for this spatial frequency.

Application N°1

using a recorded correlation
to locate a point source in the sky



The 2-element interferometer : Application: position of a source

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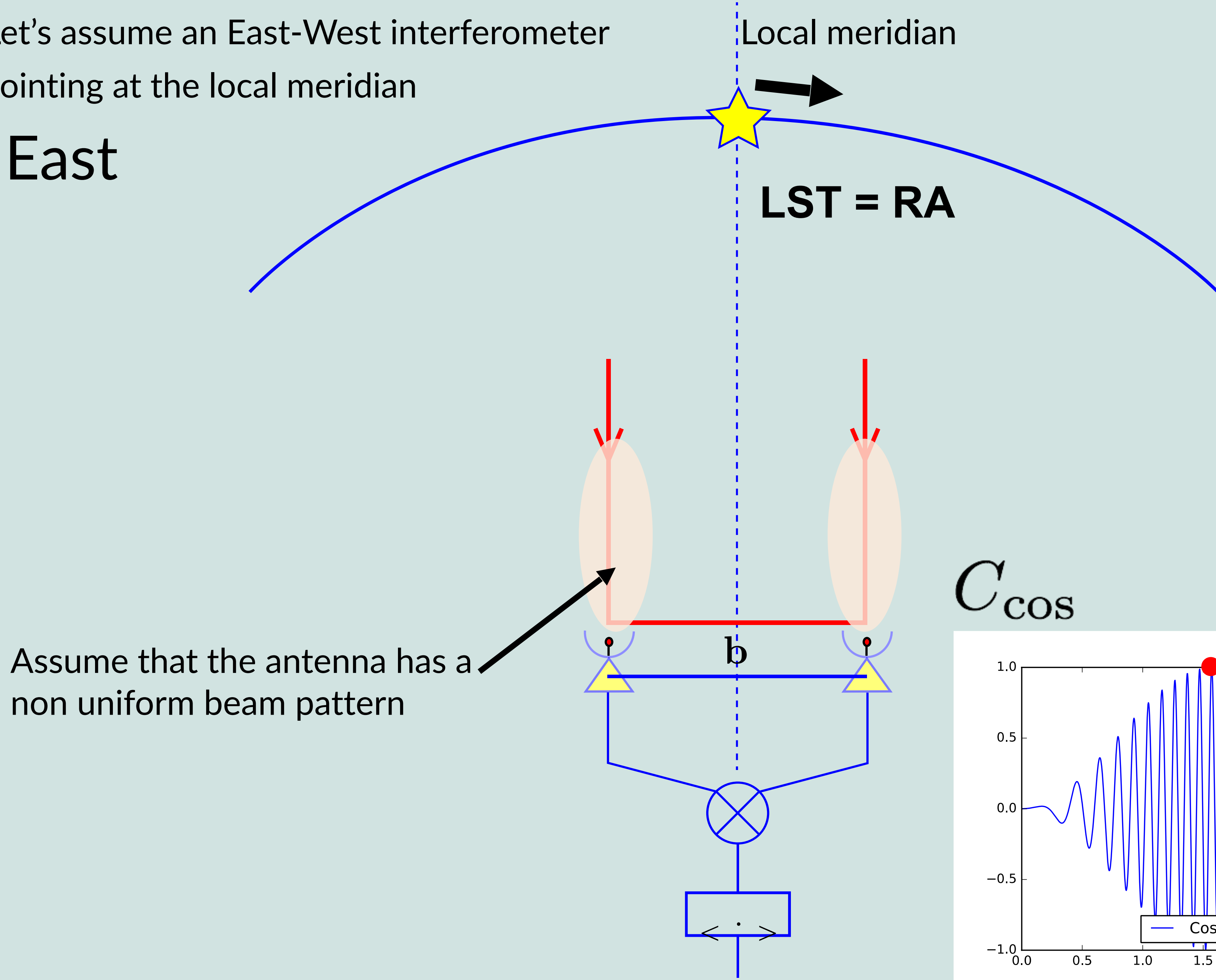
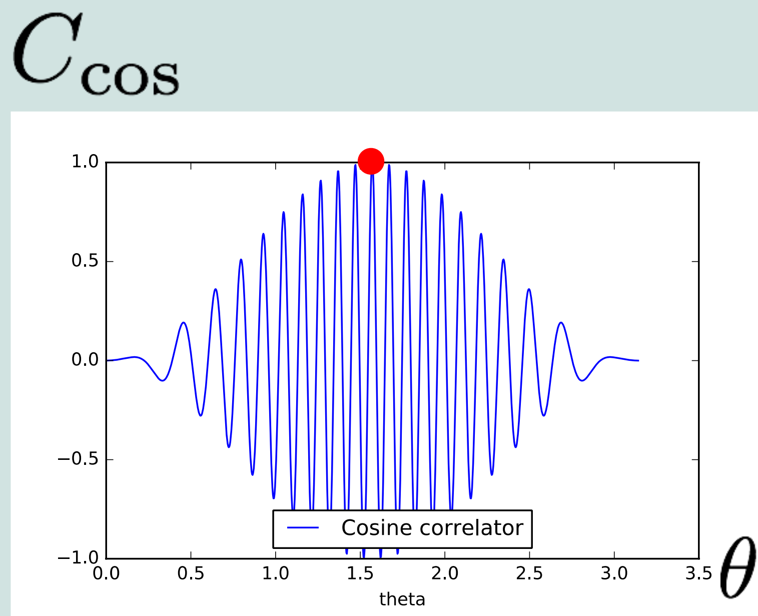


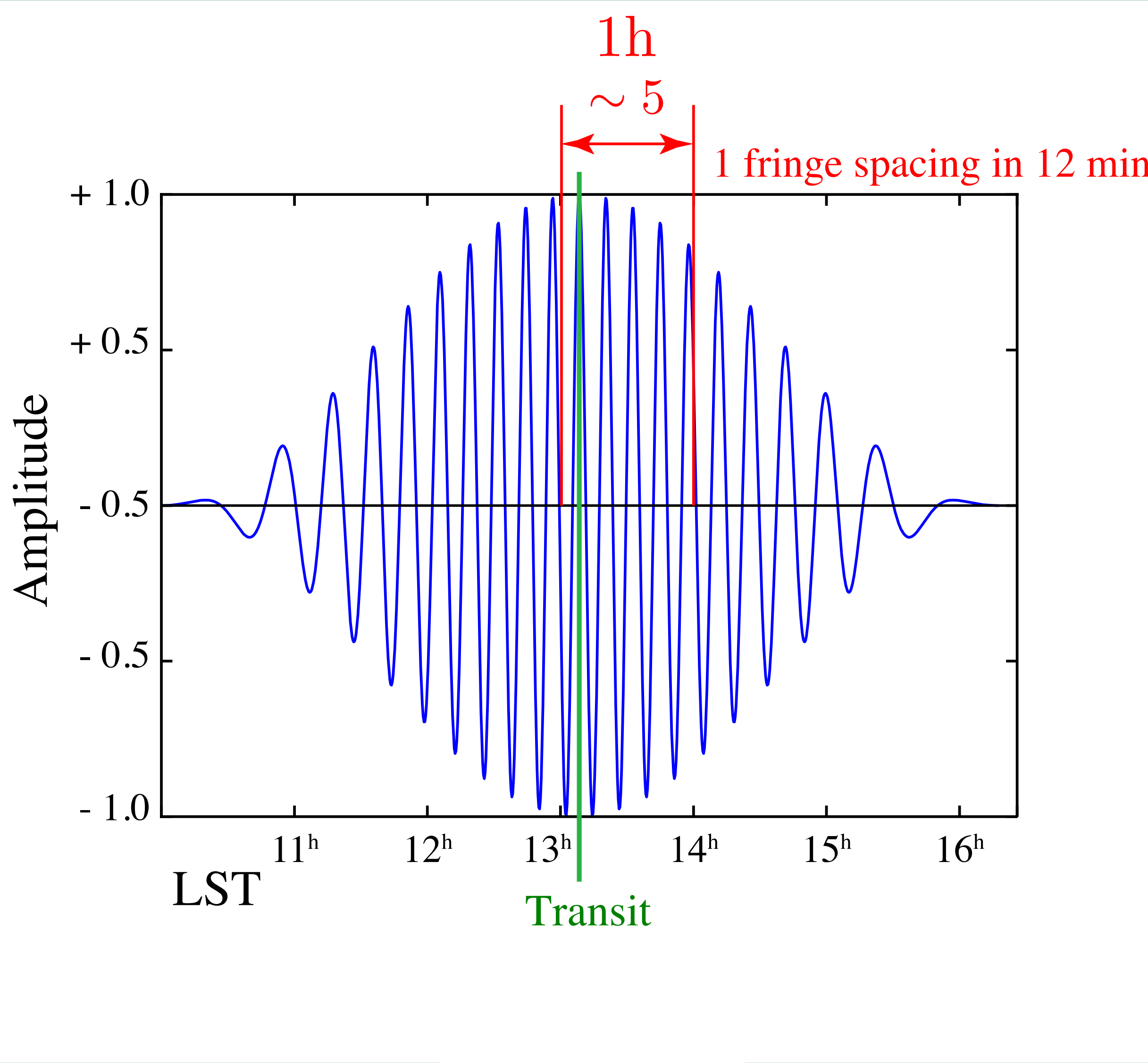
Assume that the antenna has a non uniform beam pattern



The 2-element interferometer : Application: position of a source

Estimation of α

High precision measurement of the transit time



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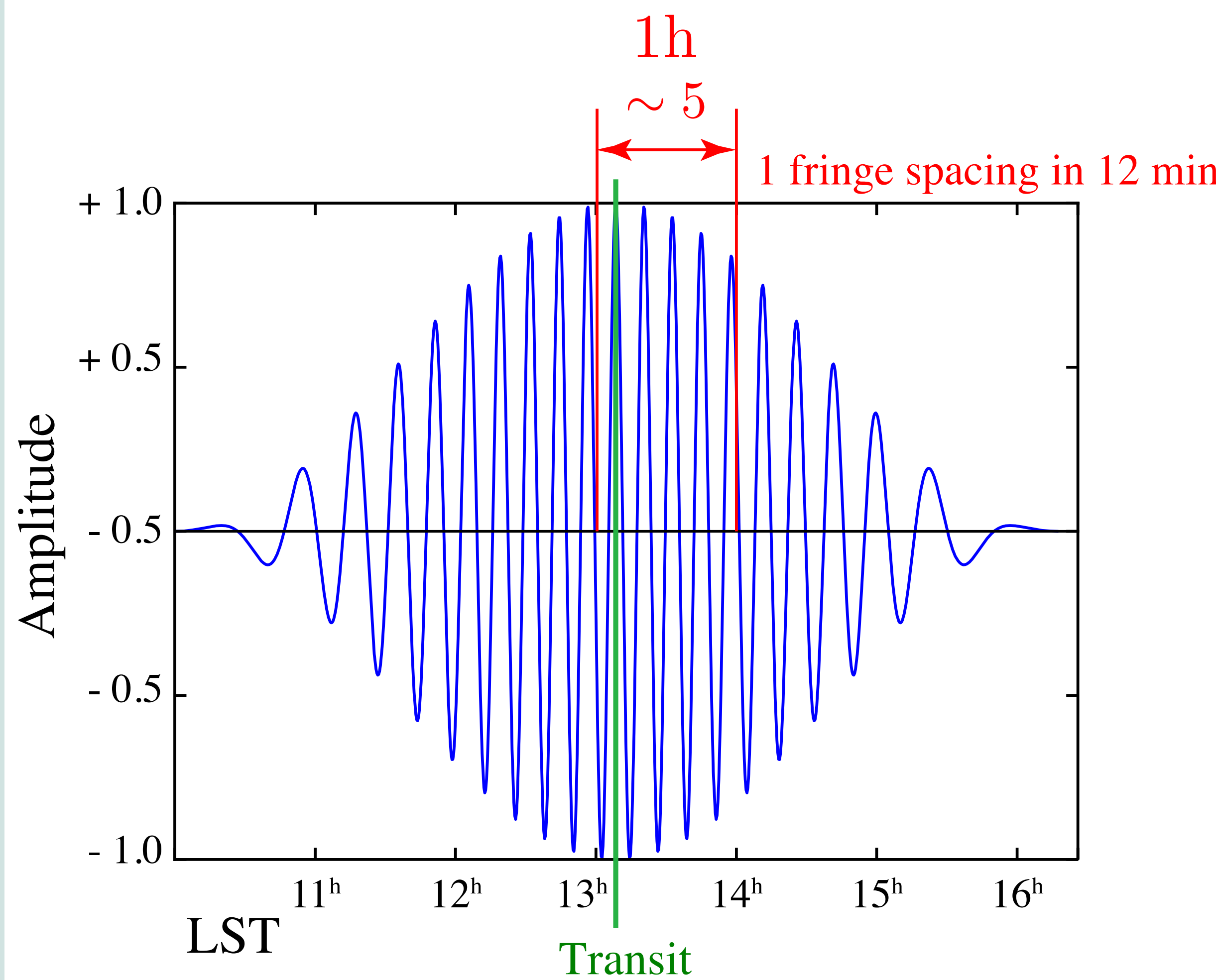

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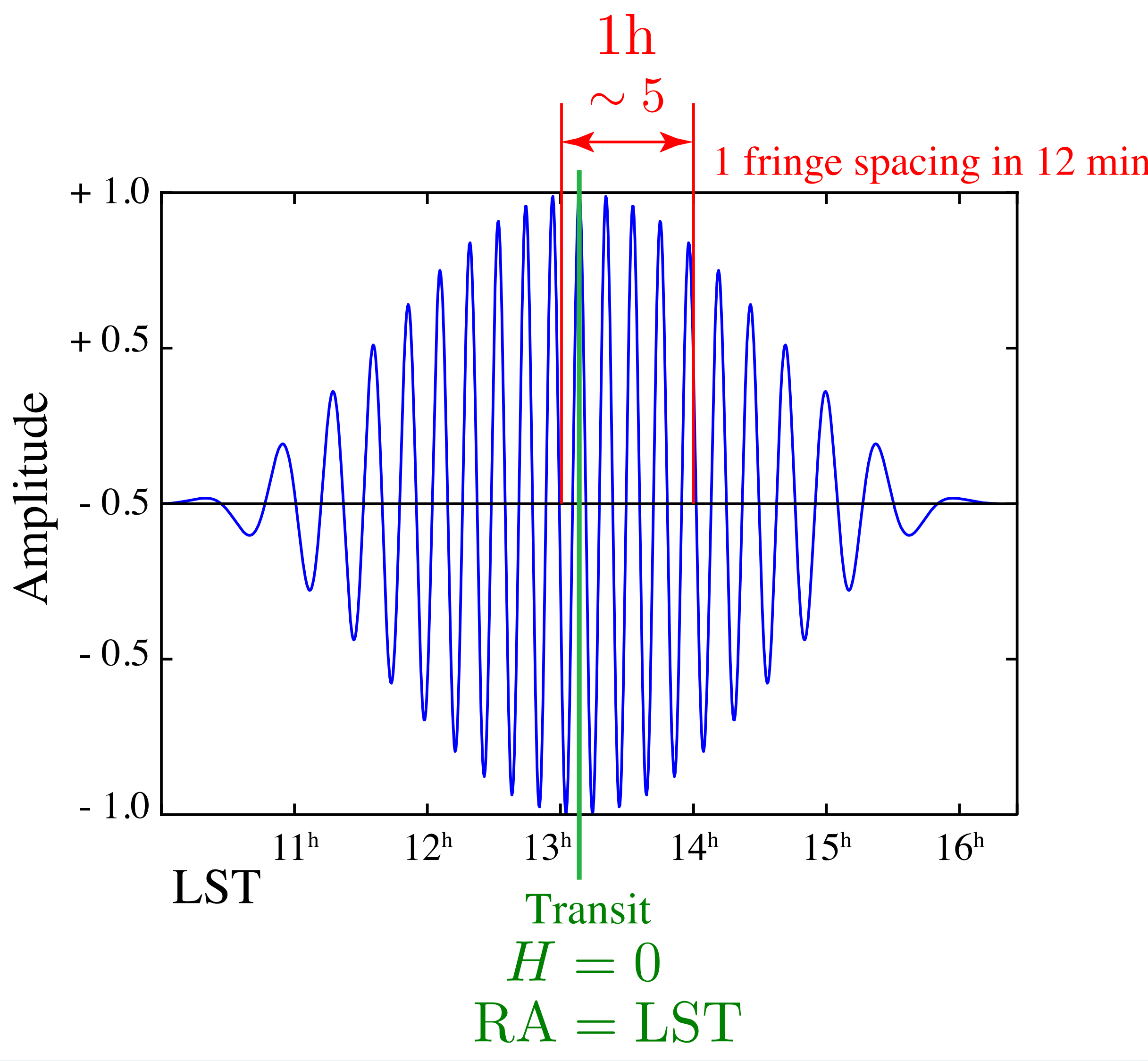

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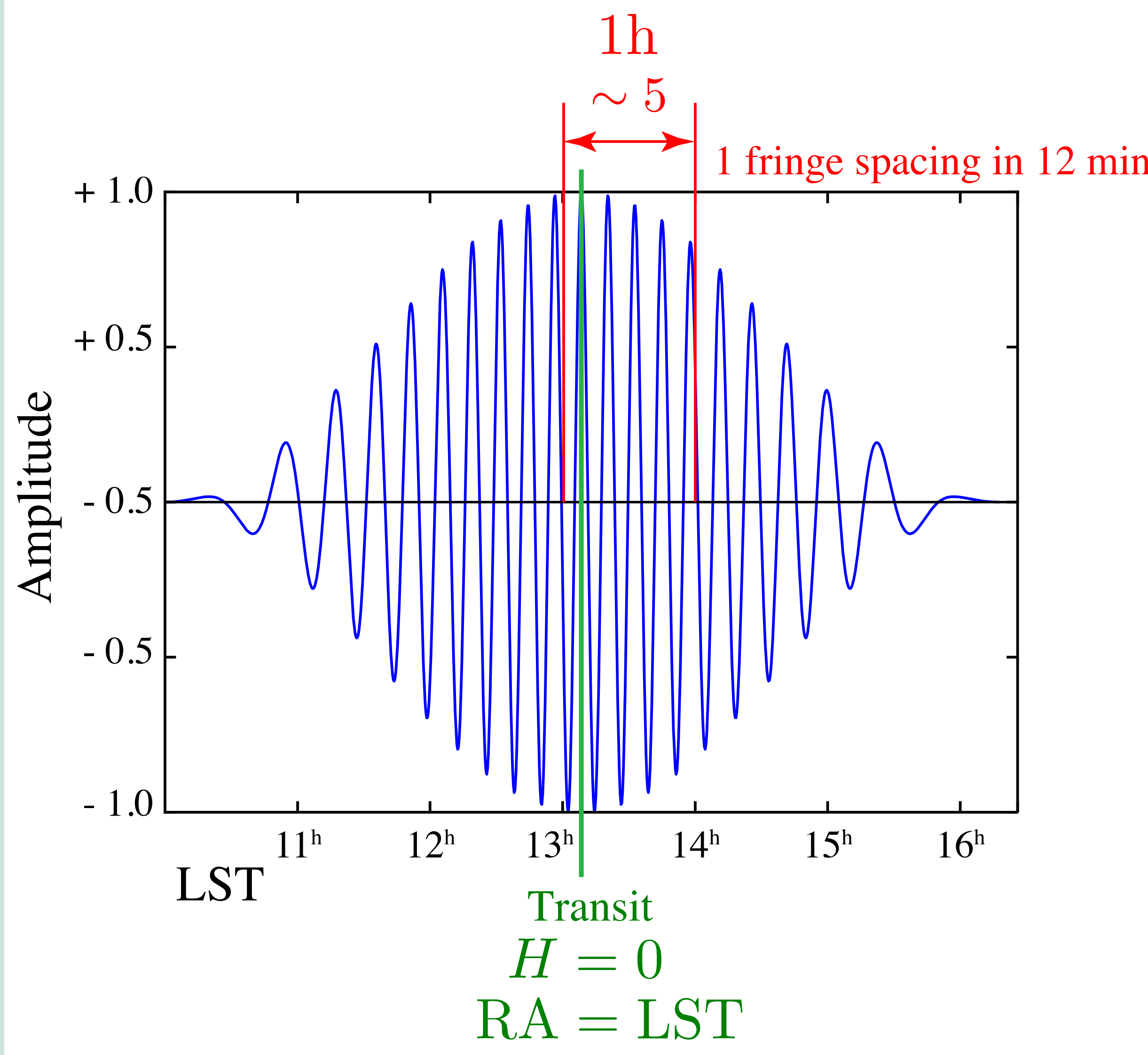

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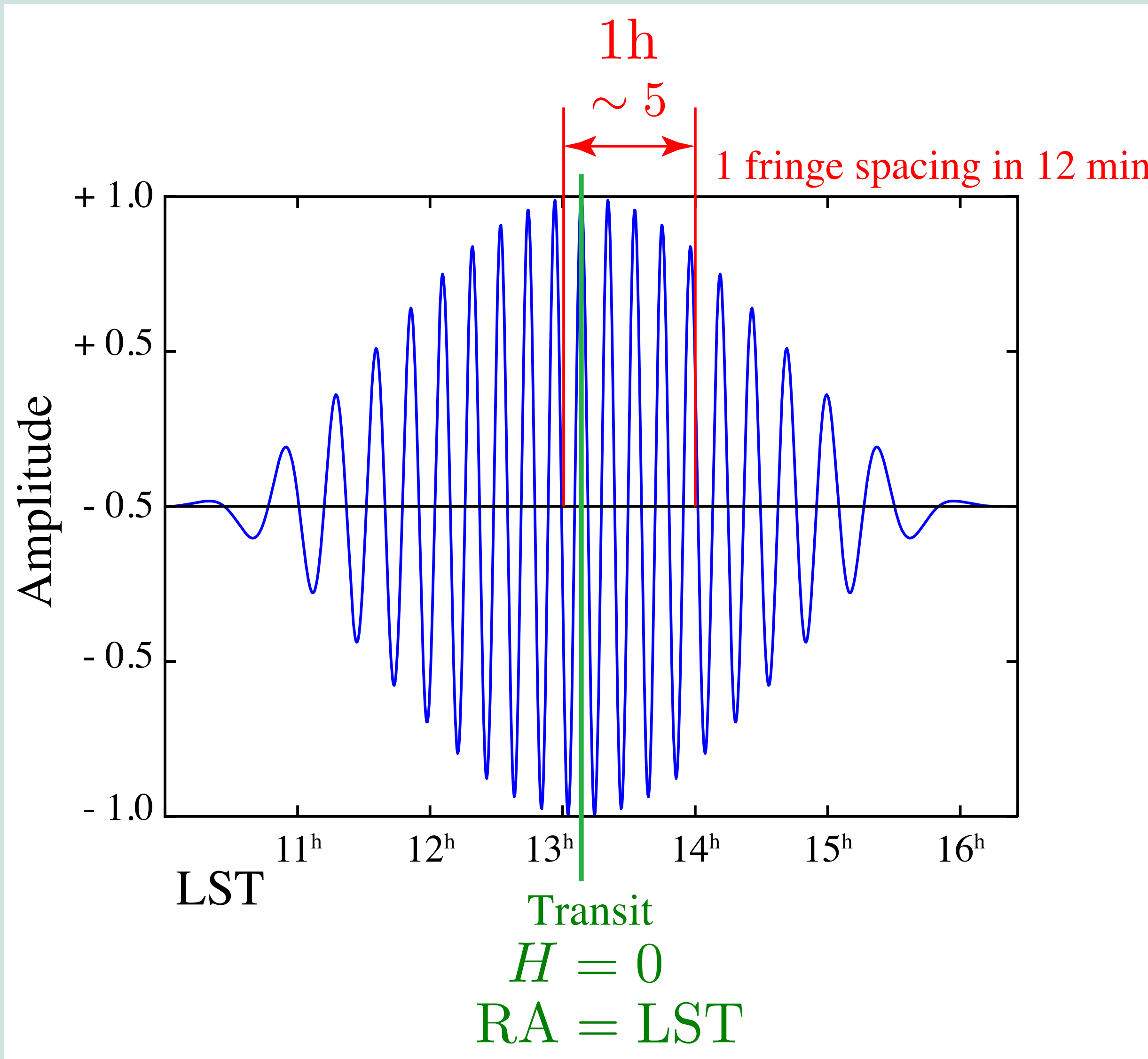

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Here, we can estimate graphically that $\alpha \sim 13^{\mathrm{h}}07^{\mathrm{m}}$

The 2-element interferometer : Application: position of a source

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We define the fringe spacing :

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the measured angular distance on the sky corresponding to one spatial period of the fringe pattern projected on the sky.

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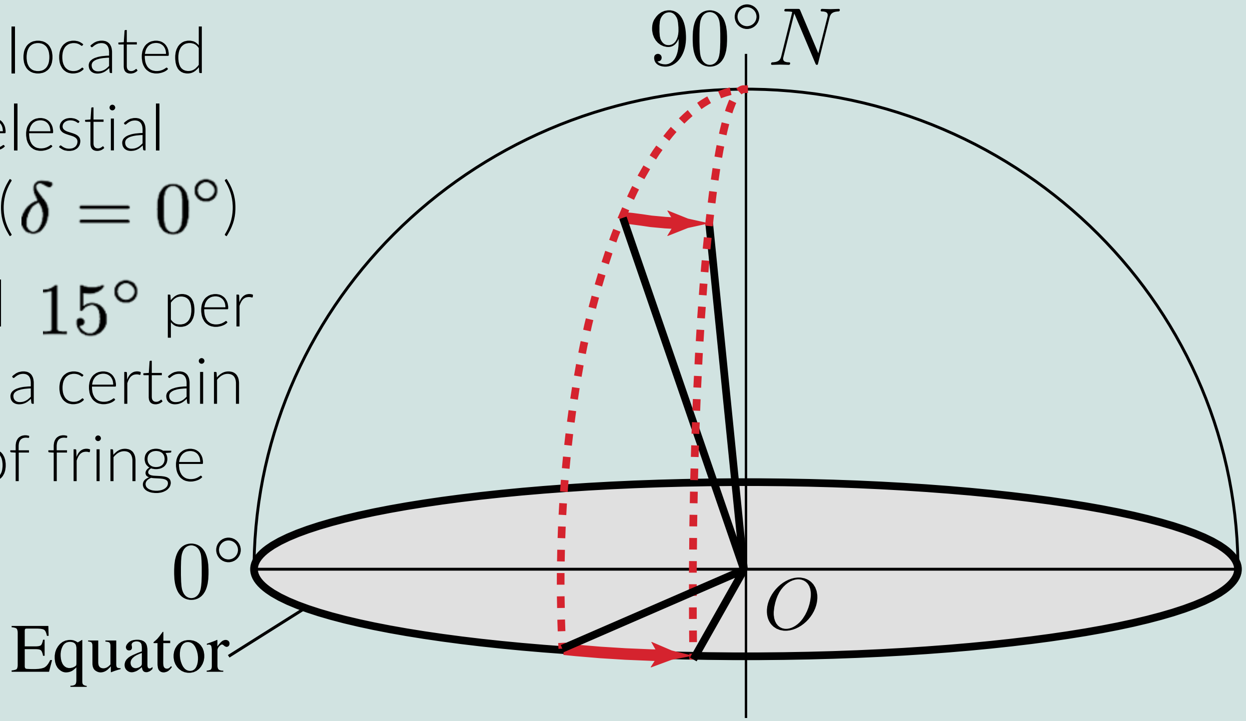
In our example:

$$\Delta l_f \sim \frac{1}{2864} \text{ rad} = 0.02 \text{ rad} \approx 1.14^\circ$$ (given)

$\frac{d\phi}{d\theta}$ **~ 12 min (~5 periods are crossed in 1h of observation)**

The 2-element interferometer : Application: position of a source

A source located on the celestial equator ($\delta = 0°$) will travel $15°$ per hour and a certain number of fringe spacings



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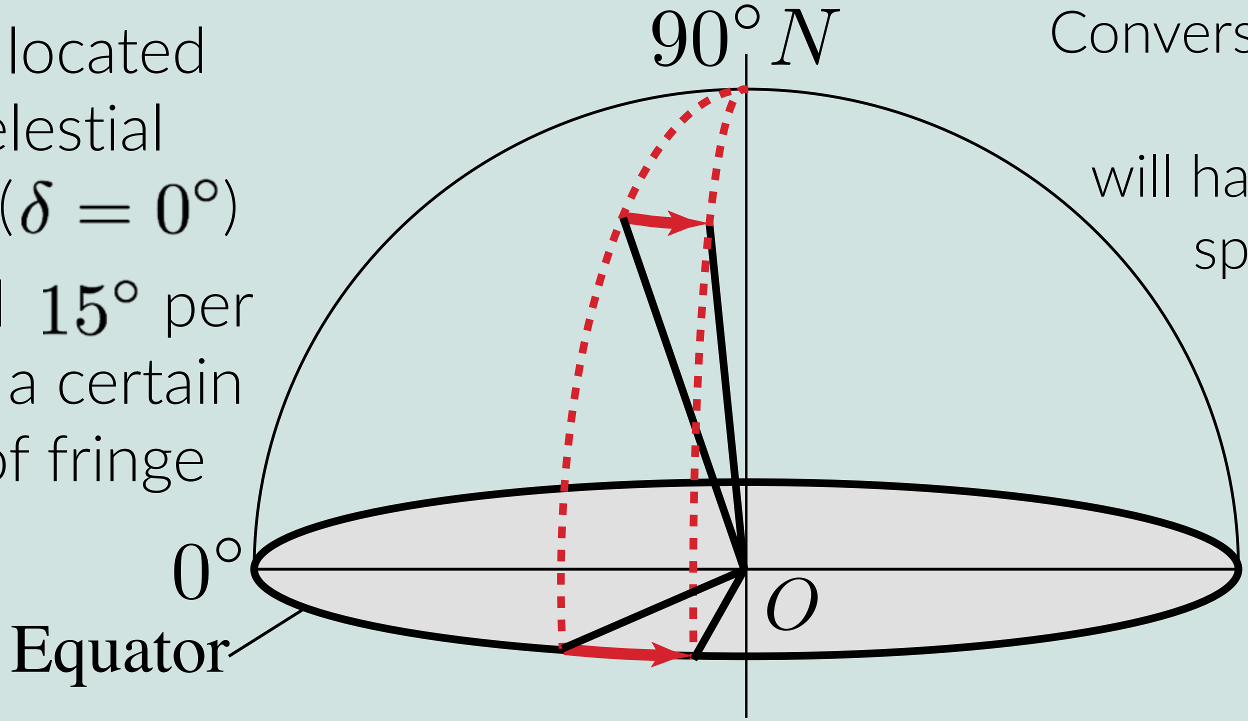

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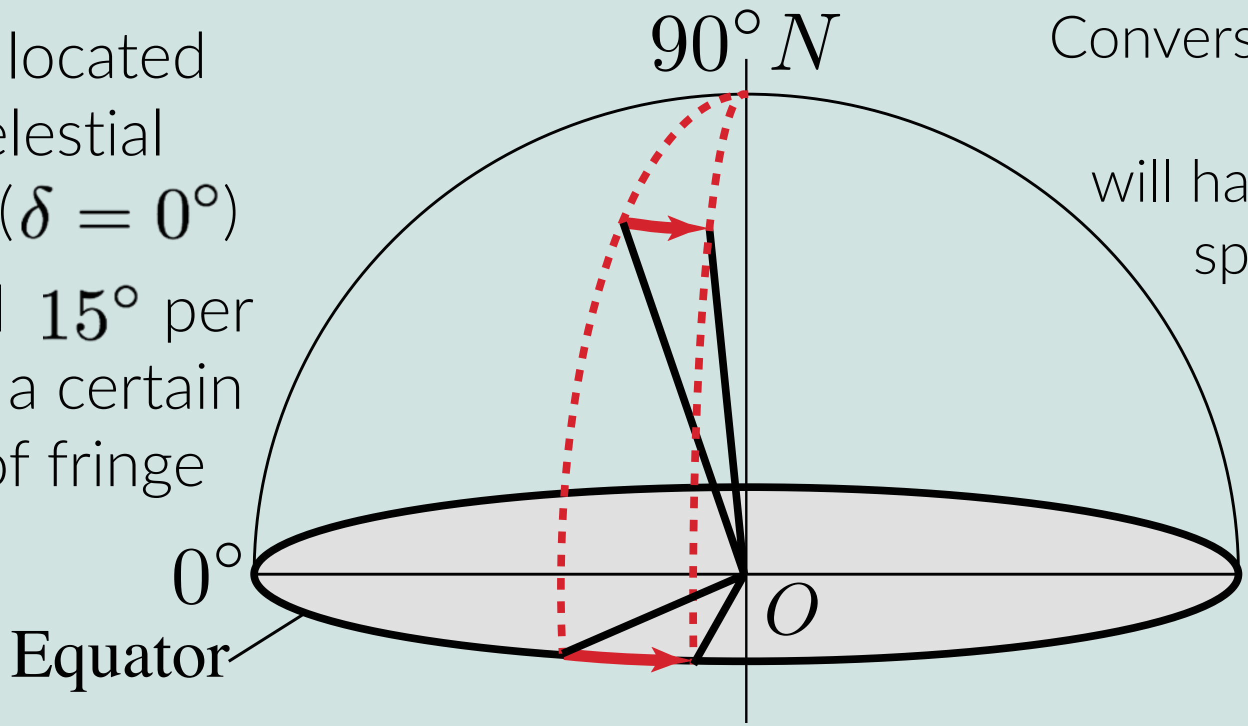

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Knowing the fringe spacing, we can derive the fringe rate at the equator with

$$\left.\frac{d\phi}{d\theta}\right|_{\text{eq}} = \frac{\Delta l_f}{15°\text{per h}} \approx 0.076h \approx 4^{\text{m}}33^{\text{s}}$$

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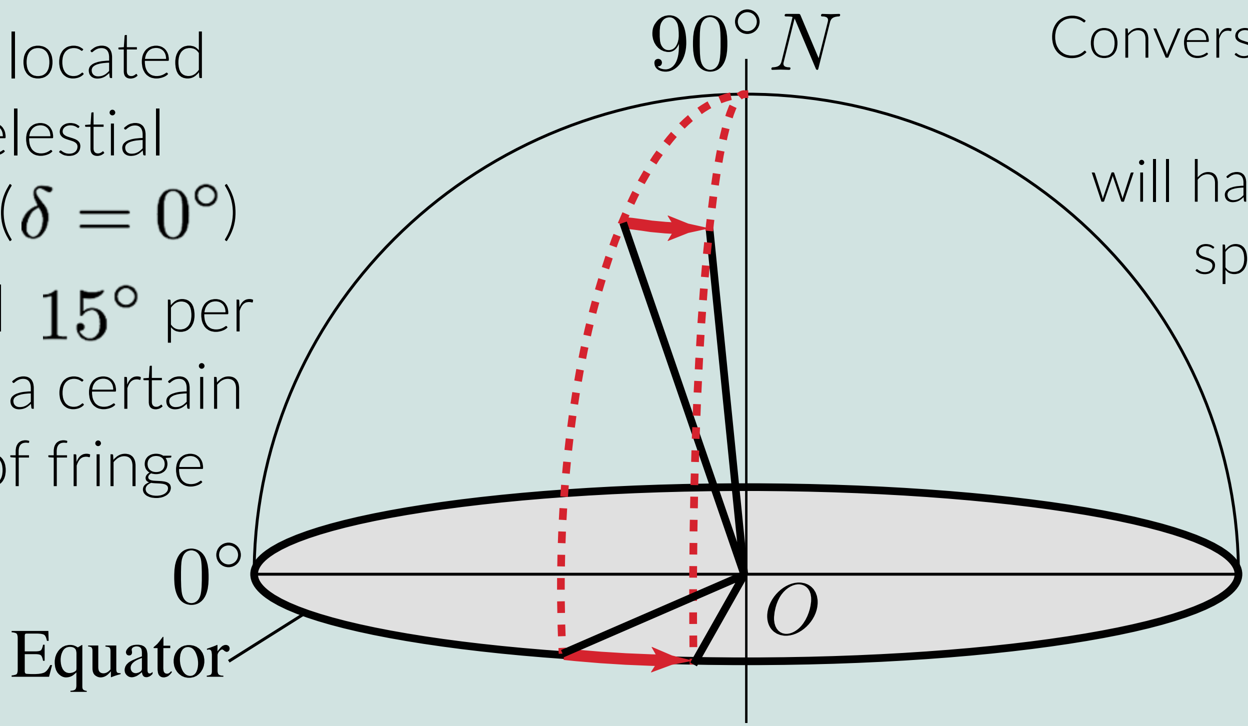

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To have an estimation of the source declination, we need to compare the fringe rate at the equator to that measured on the fringe pattern.

$$\cos\delta = \left.\frac{d\phi}{d\theta}\right|_{\mathrm{eq}} \Big/ \left.\frac{d\phi}{d\theta}\right|_{\mathrm{mes}} = \frac{4^{\mathrm{m}}33^{\mathrm{s}}}{12^{\mathrm{m}}} = 0.3825 \leftrightarrow \delta \approx 67.7°$$

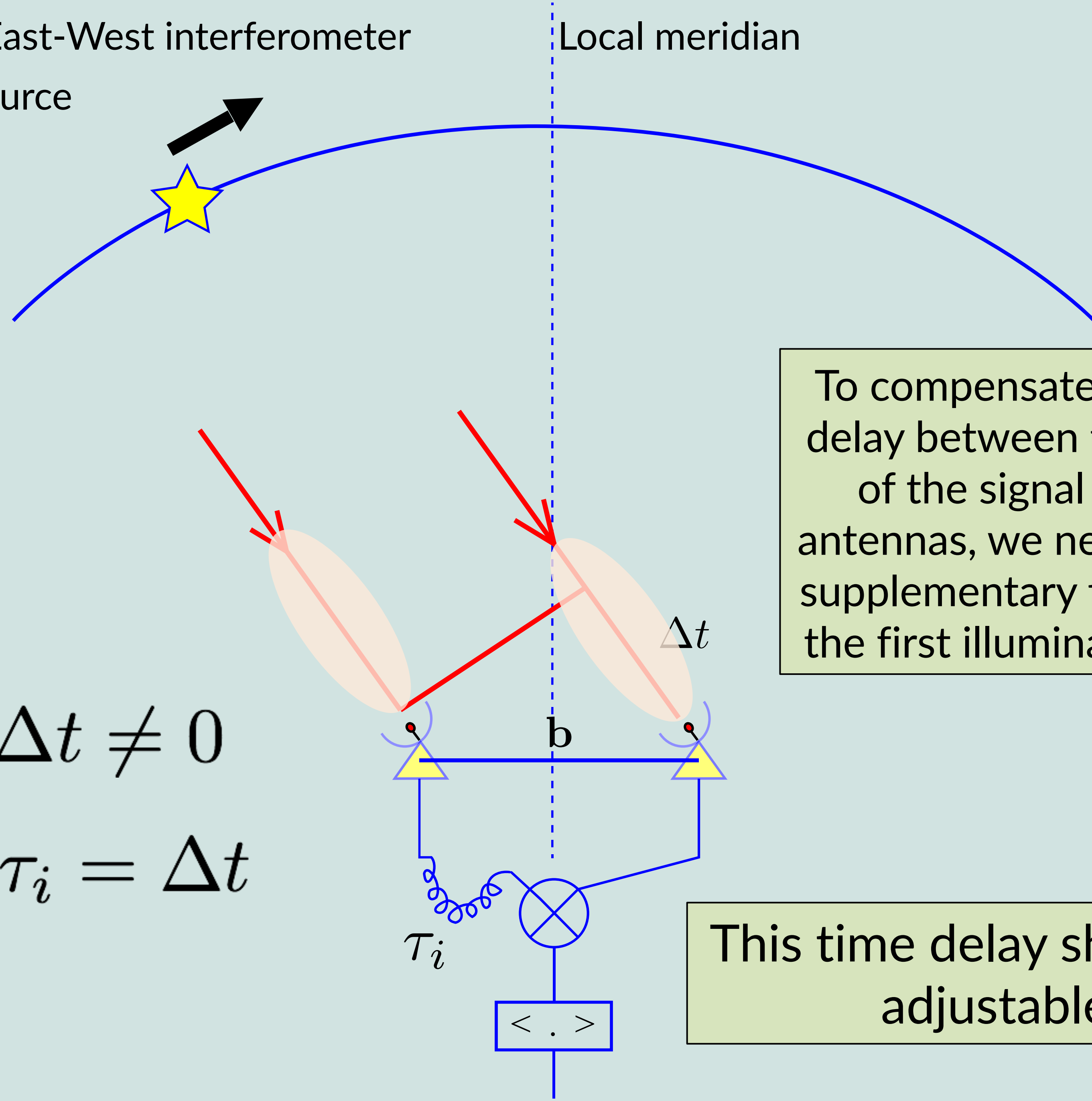
Π interferometer

Delay tracking
&
Phase center

The 2-element interferometer : Delay tracking

Let's assume an East-West interferometer

Pointing at the source



To compensate for the time delay between the reception of the signal at the two antennas, we need to insert a supplementary time delay for the first illuminated antenna.

$$\Delta t \neq 0$$

East

West

$$\tau_i = \Delta t$$

This time delay should be adjustable

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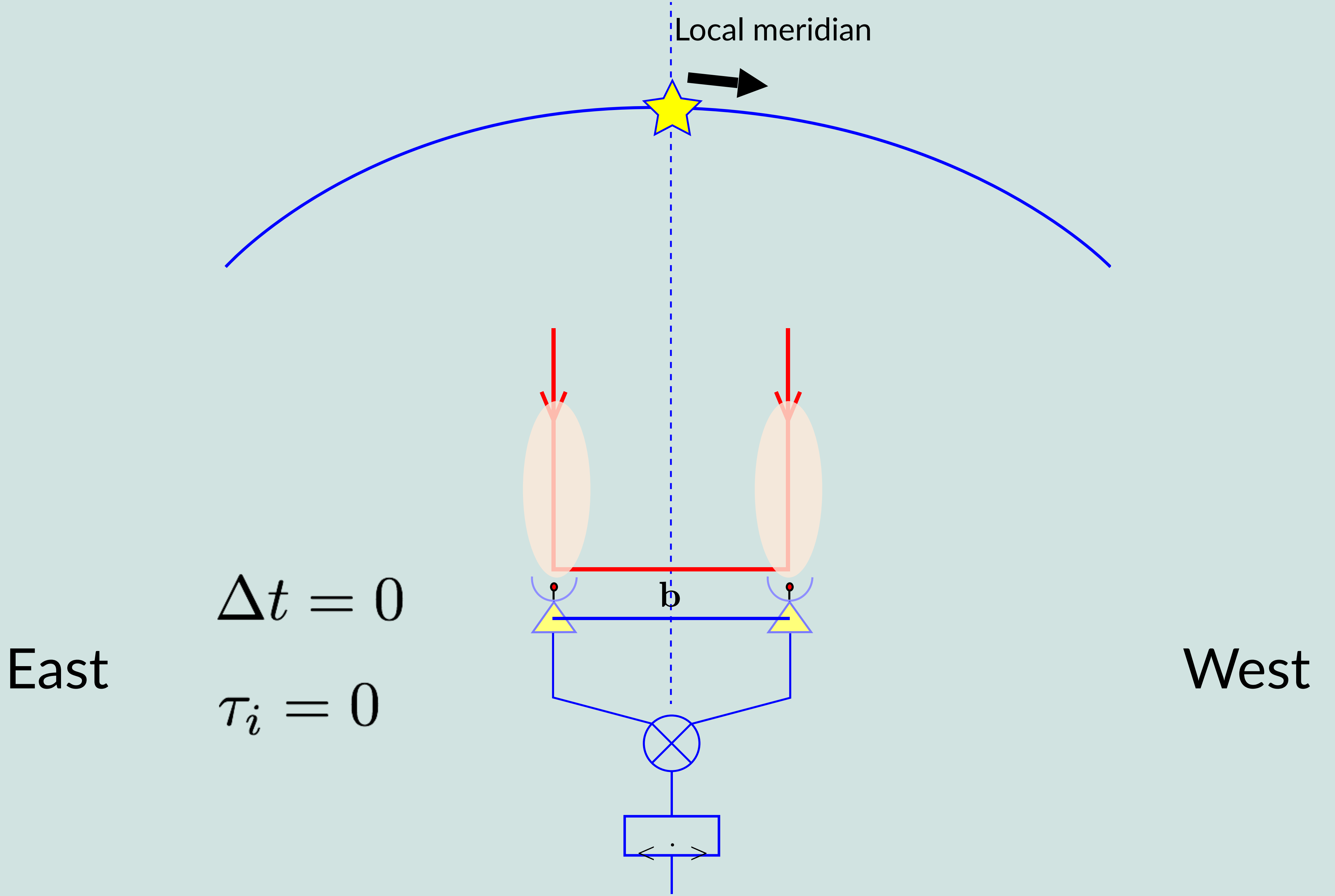

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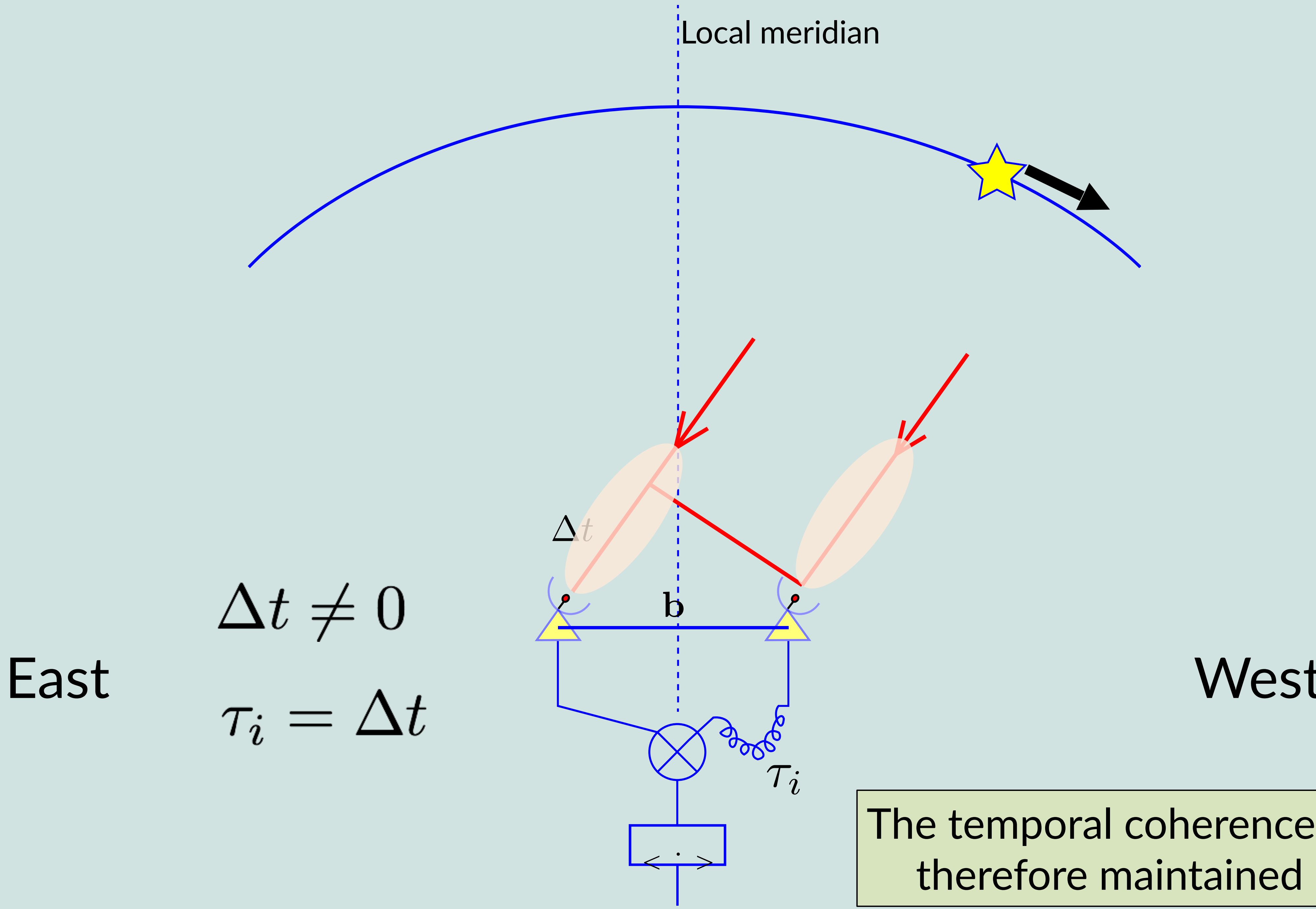

The temporal coherence is therefore maintained

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The particular direction which is tracked, defines a point on the celestial sphere, called **the phase center**

The 2-element interferometer : Delay tracking

The delay tracking is the **continuous correction of the delay between the two receivers.**

Previously, the fringe pattern has a dependence in θ $$C_{\cos} = \frac{V_0^2}{2} \cos \omega\tau$$

The delay compensation « removes » this dependence, so that for the « pointed » direction only, the correlation is always maximal $\tau_g = 0$

In this case, the antennas are also tracking the source, facing always the same antenna response to it.

The particular direction which is tracked, defines a point on the celestial sphere, called **the phase center**

In the following, we will define other directions, with respect to this phase center

using the direction cosines

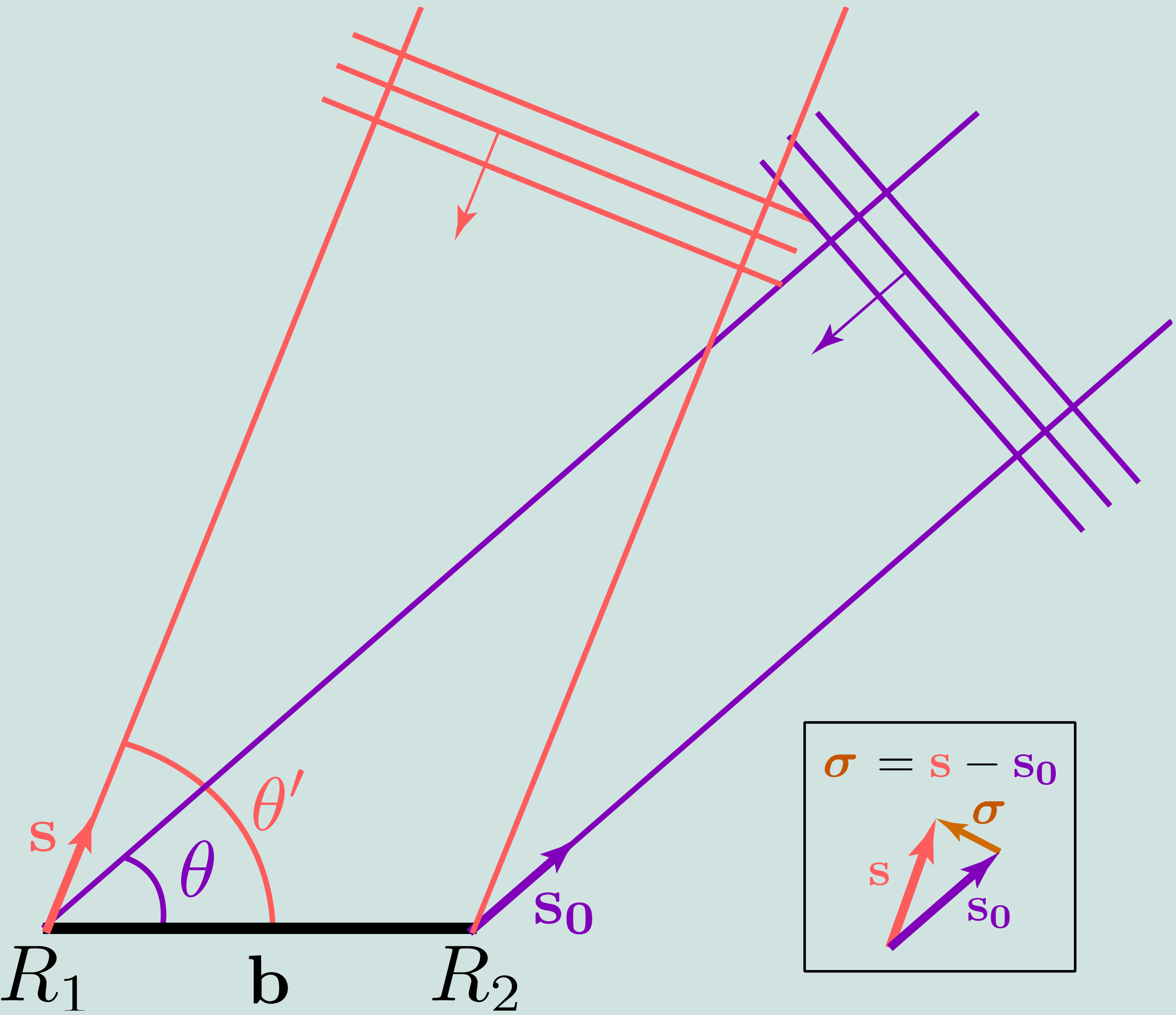
Multiple directions

the (l,m) plane

The 2-element interferometer : multiple directions

If the same 2 receivers are illuminated by two spatially incoherent waves

Each signal will generate its own delay and its own projected baseline



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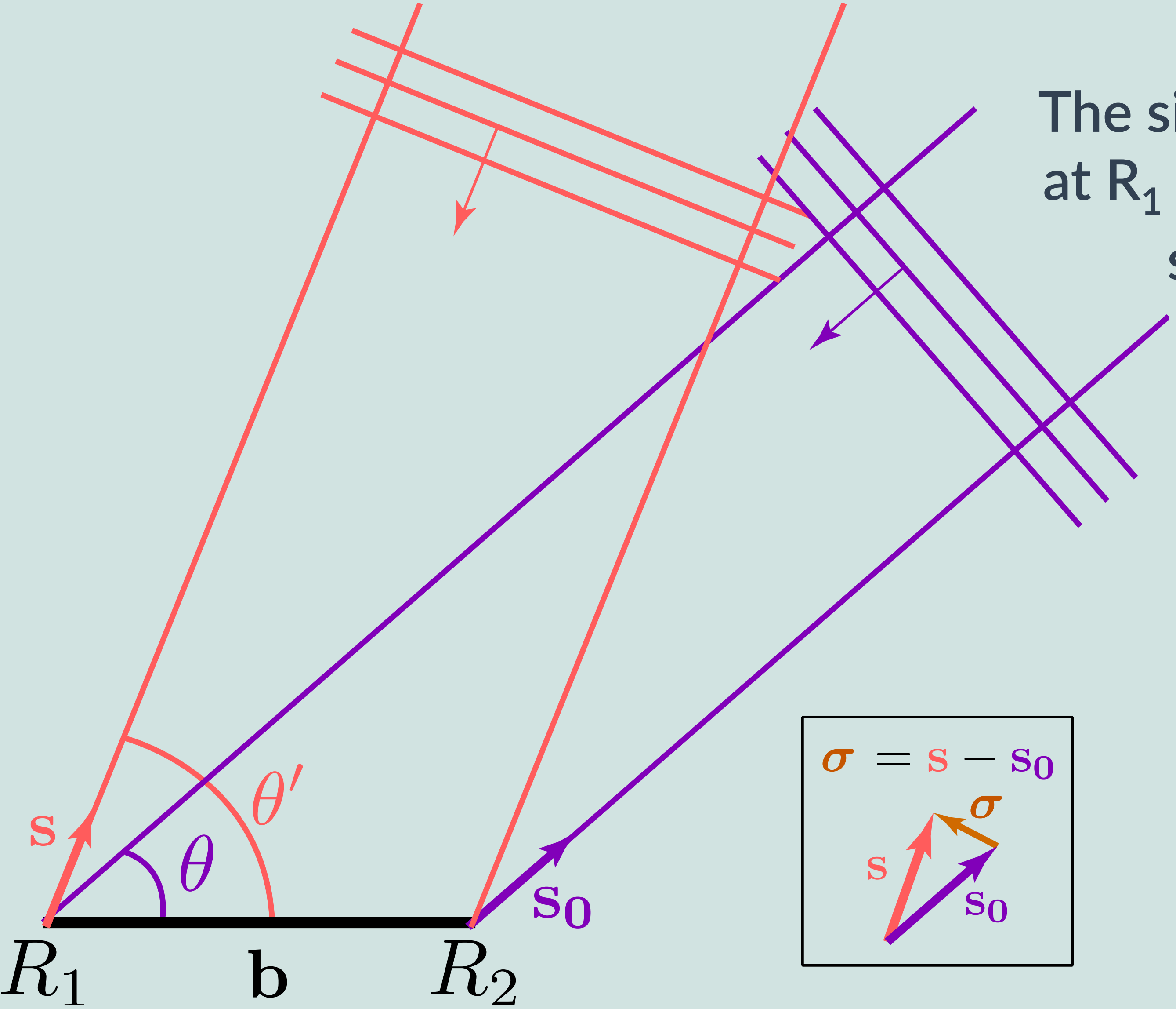

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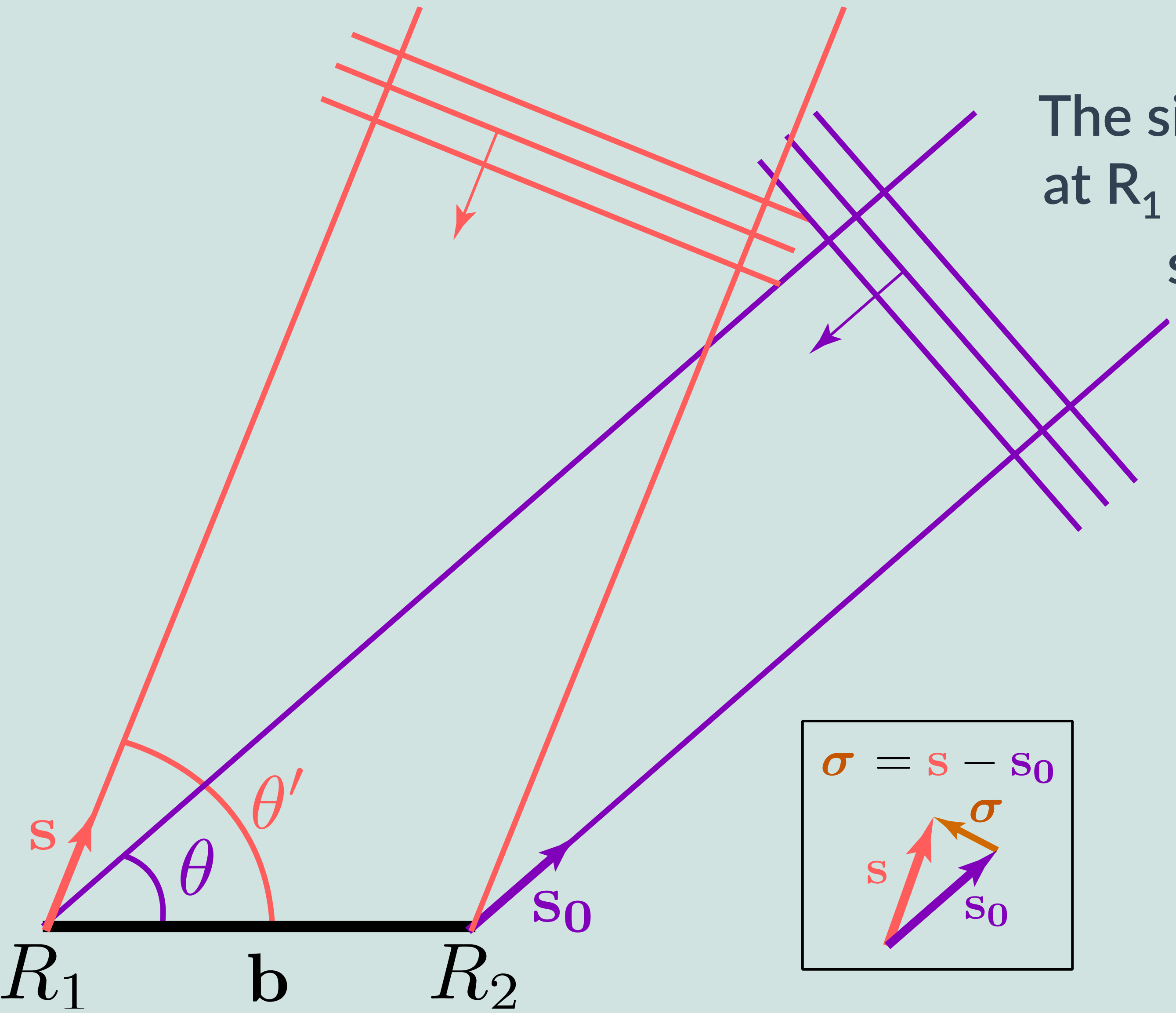

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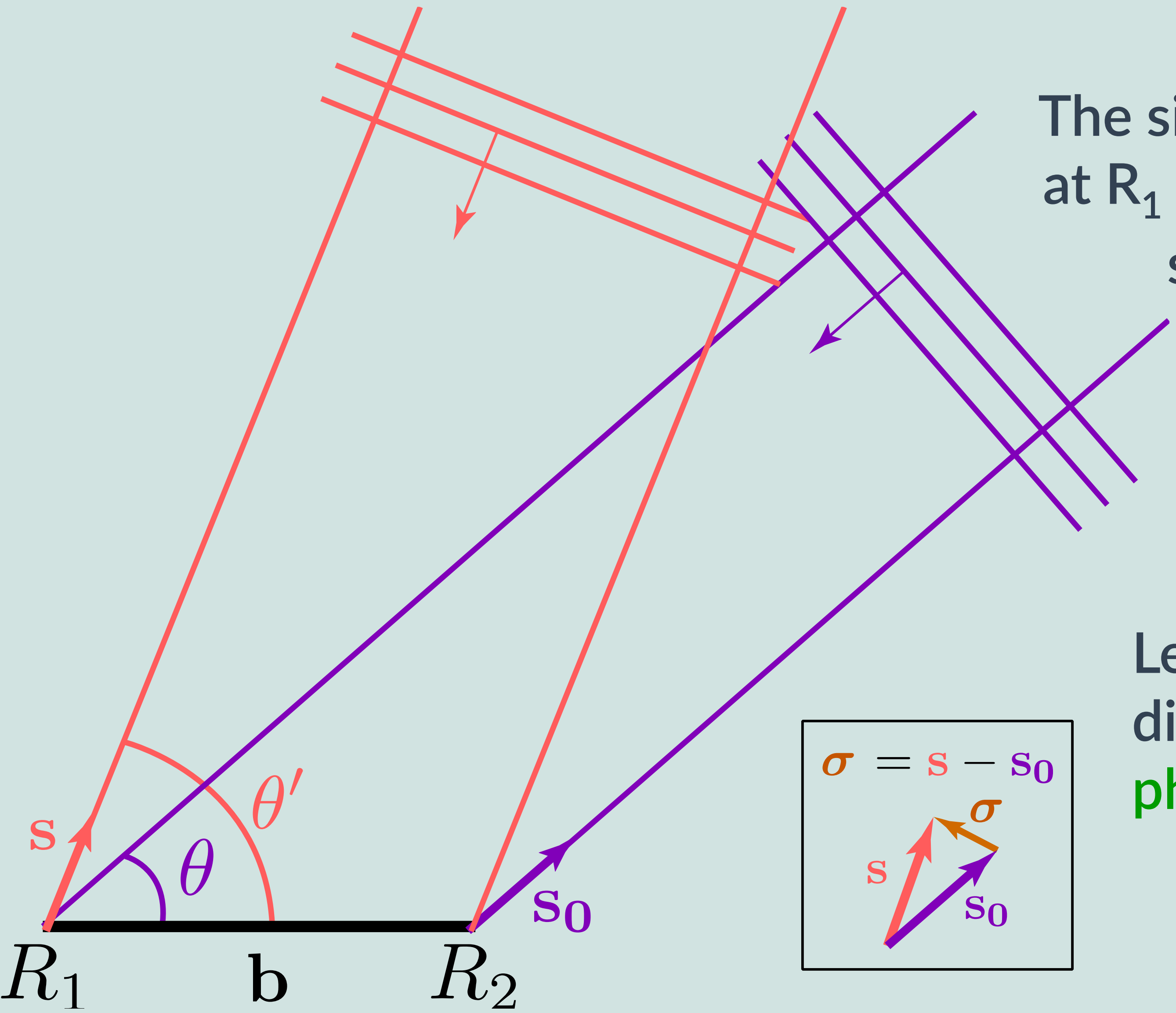

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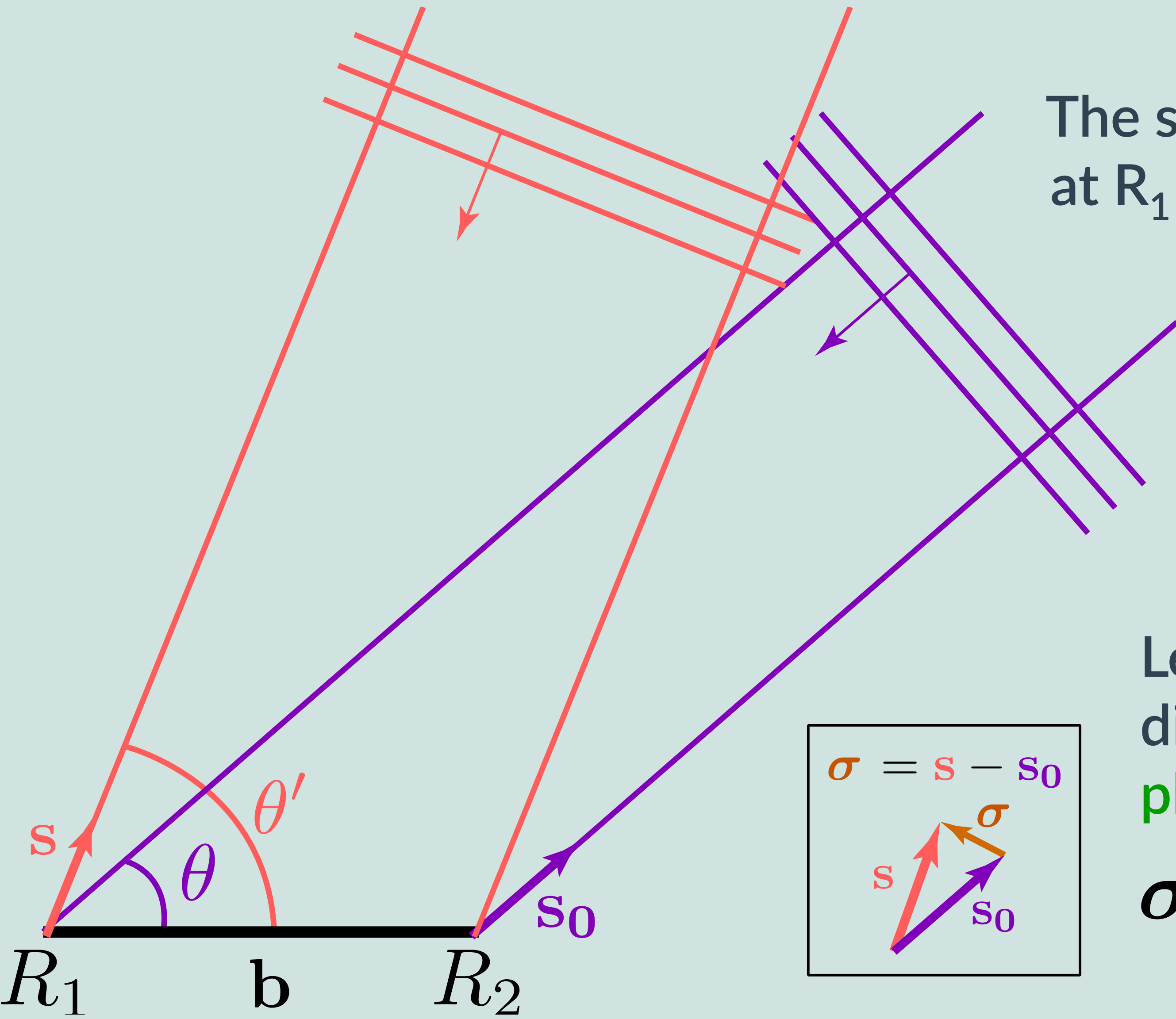

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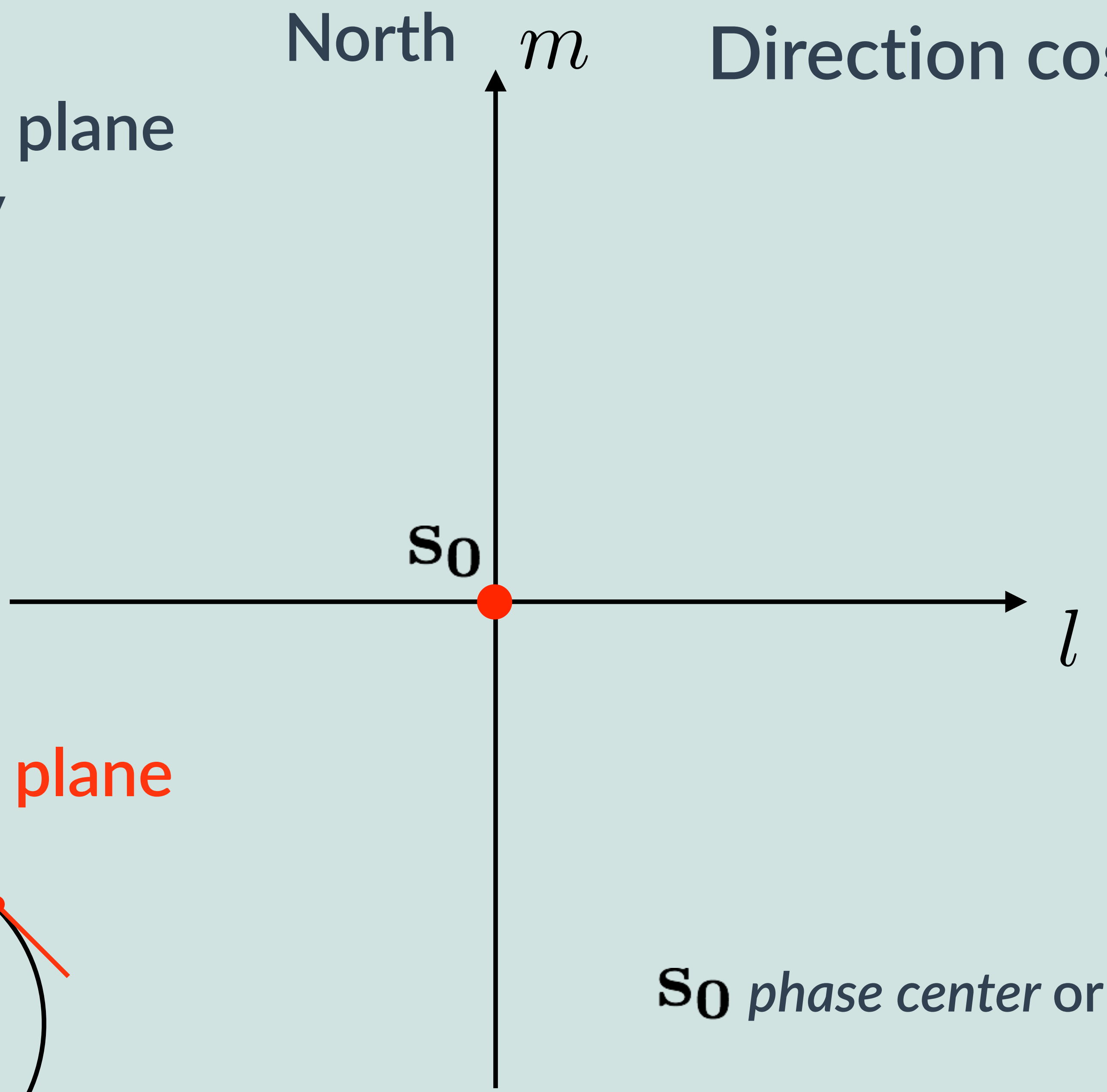
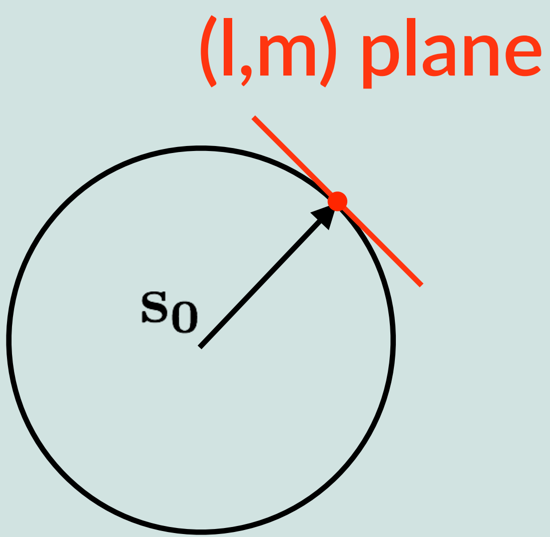
$\boldsymbol{\sigma}$ a position vector on the celestial sphere

The 2-element interferometer : multiple directions

Projected plane of the sky

Direction cosines





$\mathbf{s_0}$ *phase center* or the *field center*

The 2-element interferometer : multiple directions

Direction cosines

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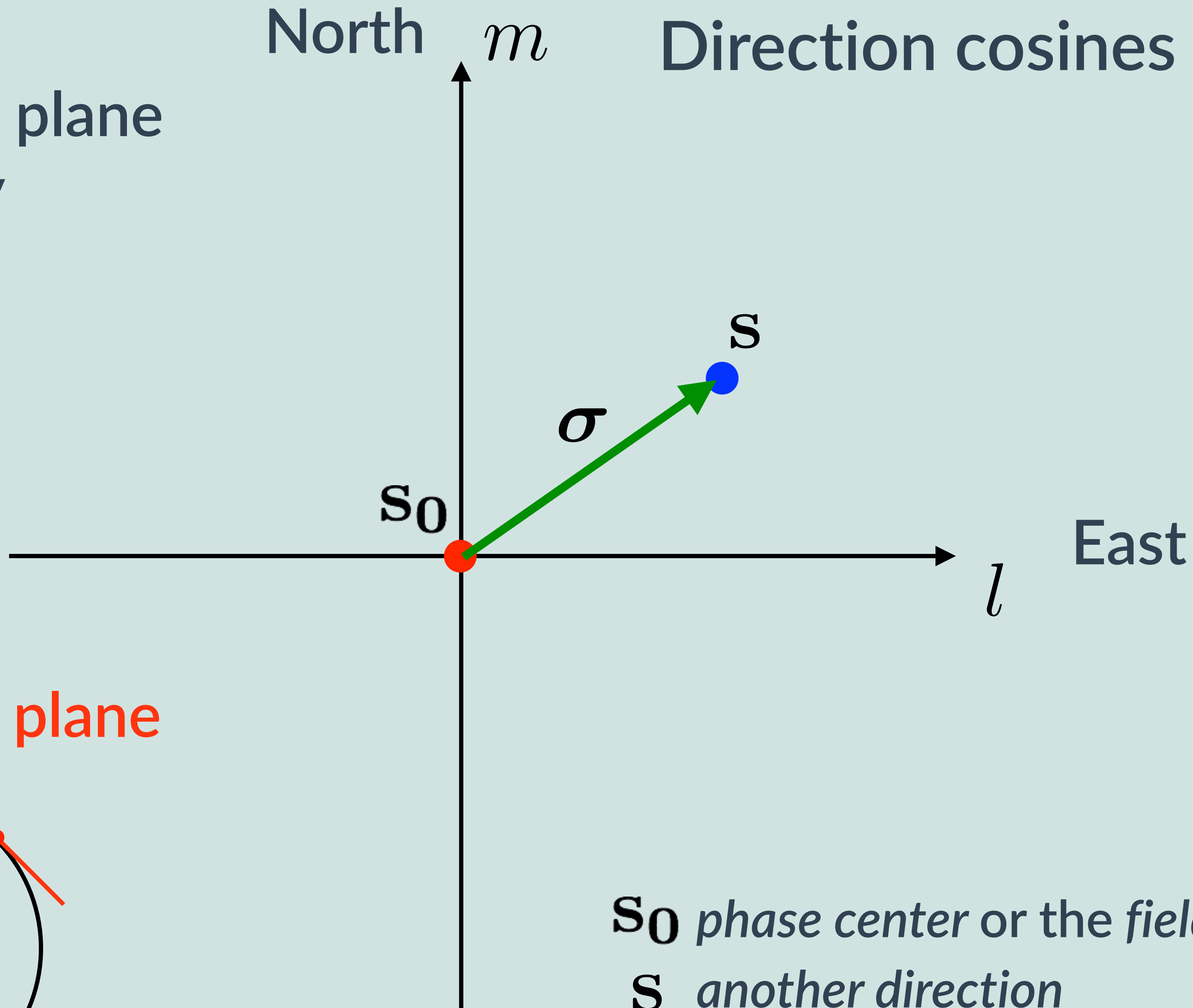

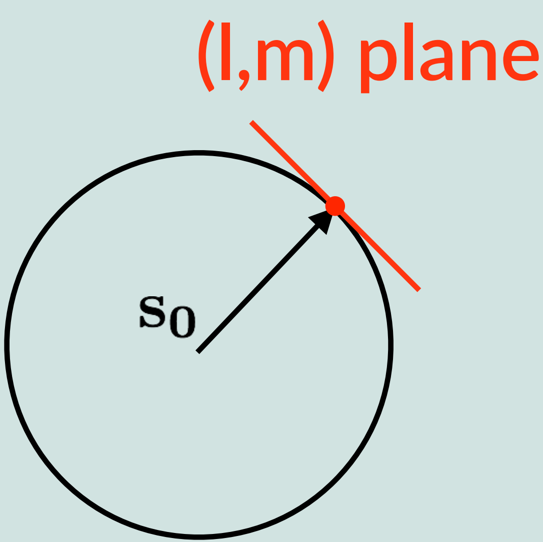

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$\mathbf{s}$ *another direction*

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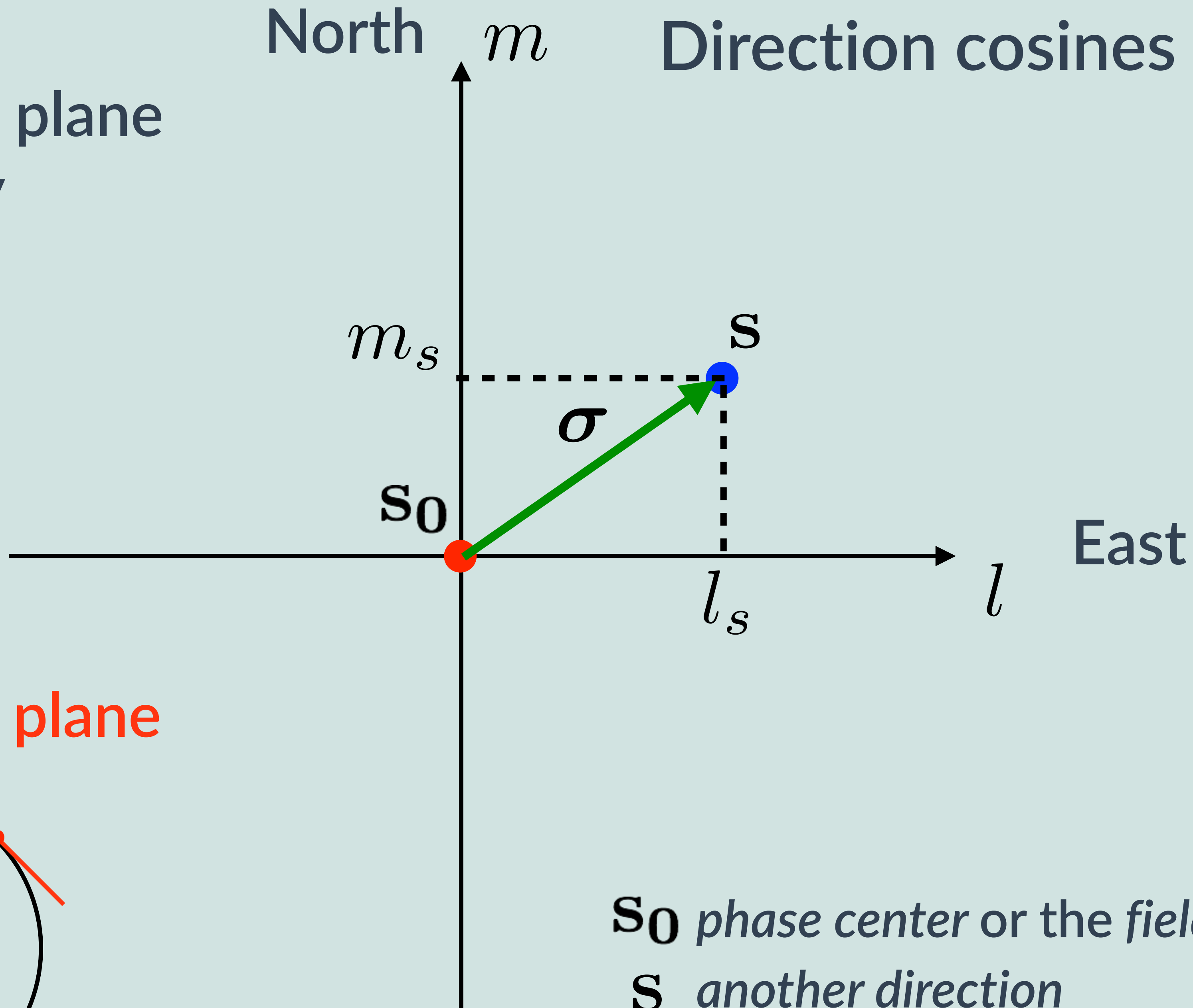

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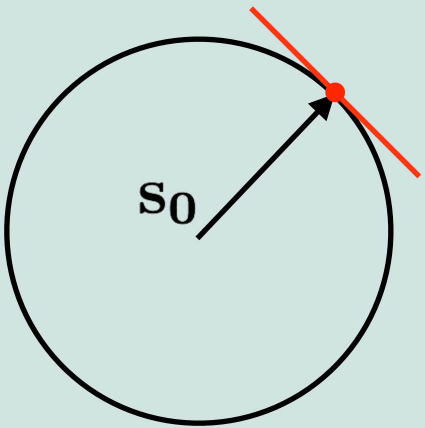

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We can express the definition of the total correlation between R_1 and R_2 to the whole sky by summing over all the observable directions

$$C_{\cos} = \int_\Omega k(\mathbf{s}) \cos(2\pi \frac{\Delta_L(\mathbf{s})}{\lambda}) d\mathbf{s}$$

with $k(\mathbf{s}) \propto \Delta\nu A(\mathbf{s}) I_\nu(\mathbf{s})$

Full derivation in the course

The 2-element interferometer : multiple directions

An interferometer measures the cross-correlation of a single incoming signal, measured by two separated probes.

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It is the sum of the sky, « as seen », through a fringe pattern looking like a venitian blind. In the following illustration, we will see how this measurement can actually inform us on the source.

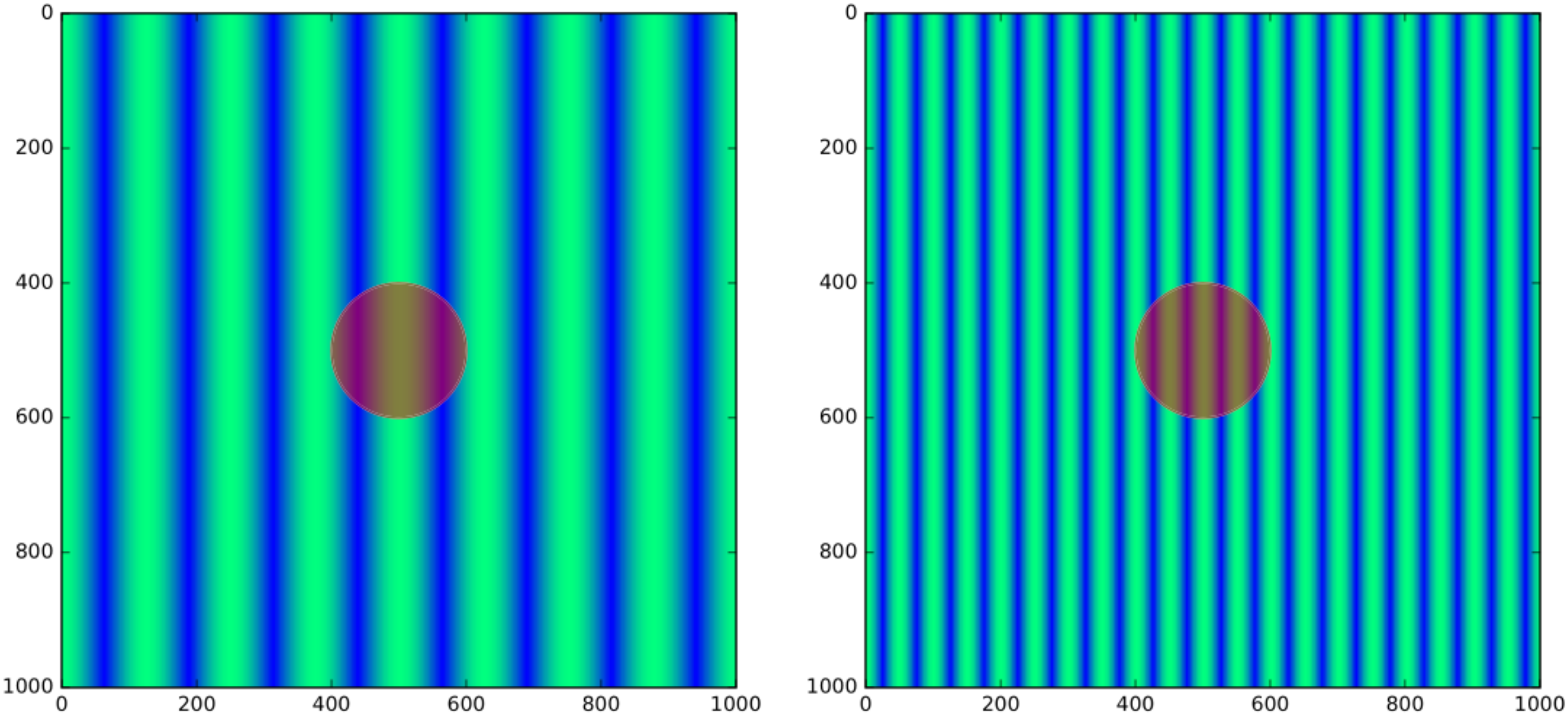
Application N°2

using a recorded correlation
to measure the structure of the source

The 2-element interferometer : structure of a source

Sky = uniform disk seen by two baselines

Disk diameter = l = 0.2



baseline 1 in direction s_0 : u = 2

Integral over disk = 12341

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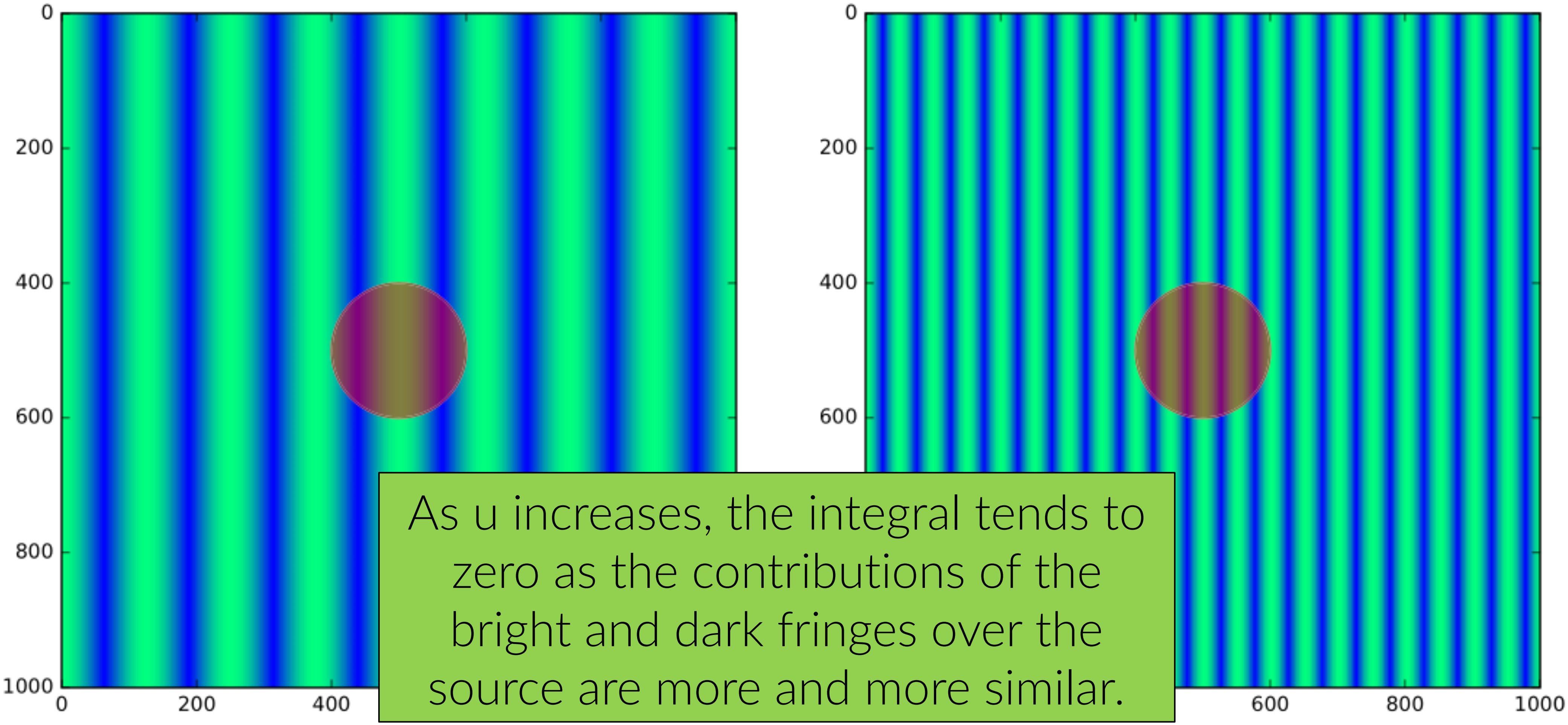

As u increases, the integral tends to zero as the contributions of the bright and dark fringes over the source are more and more similar.

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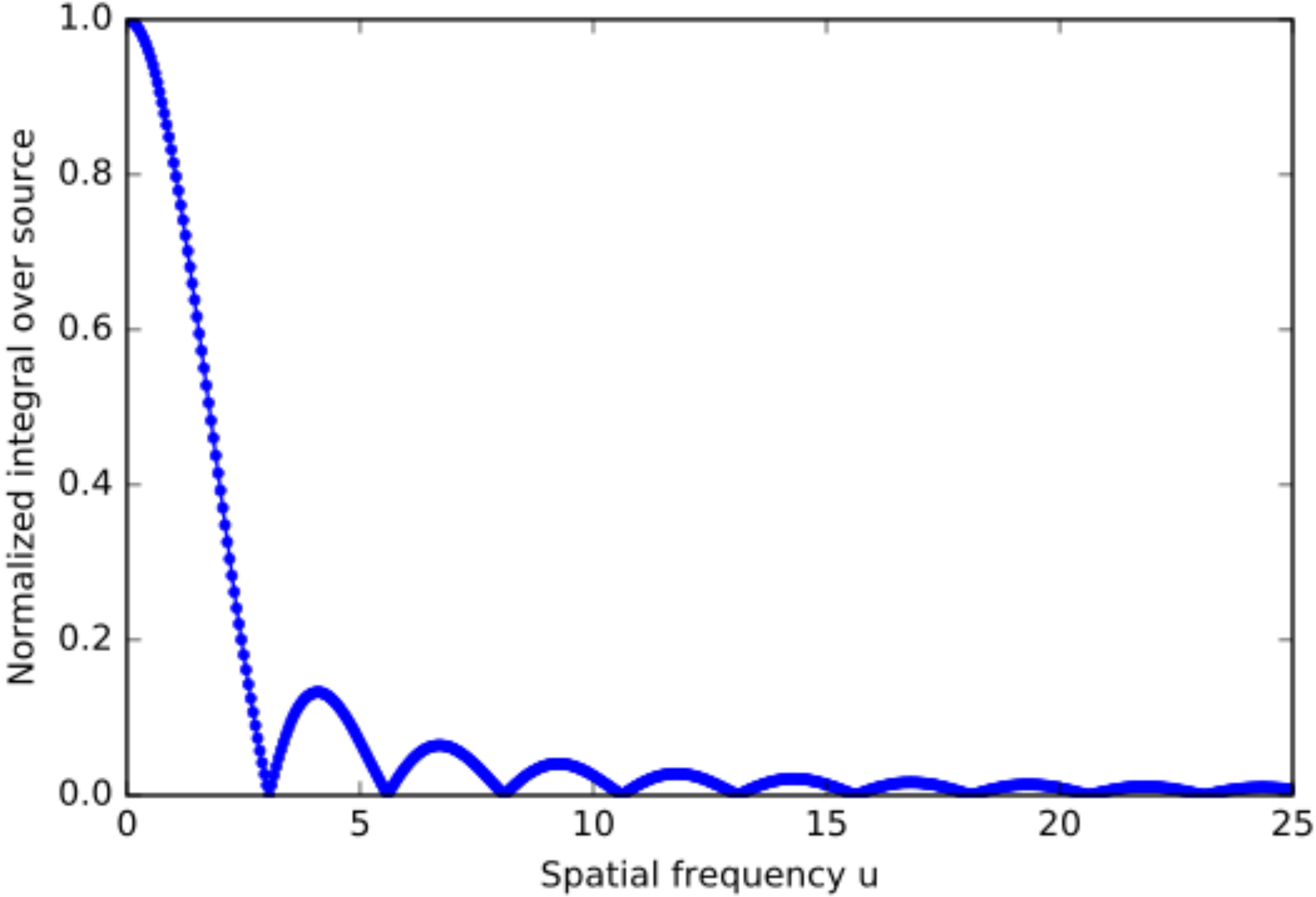
The 2-element interferometer : structure of a source

Let's compute the absolute value of the integral for different u



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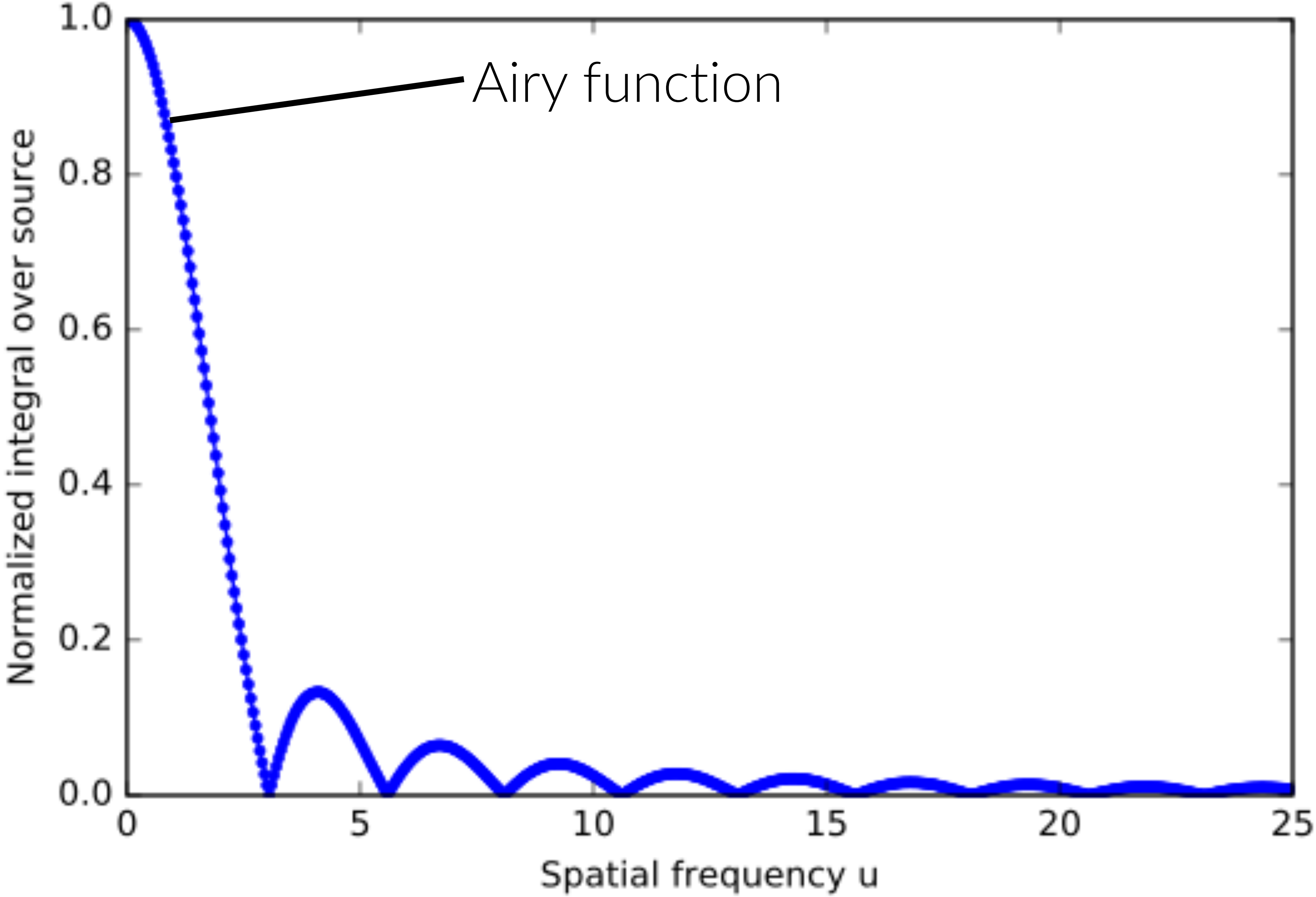

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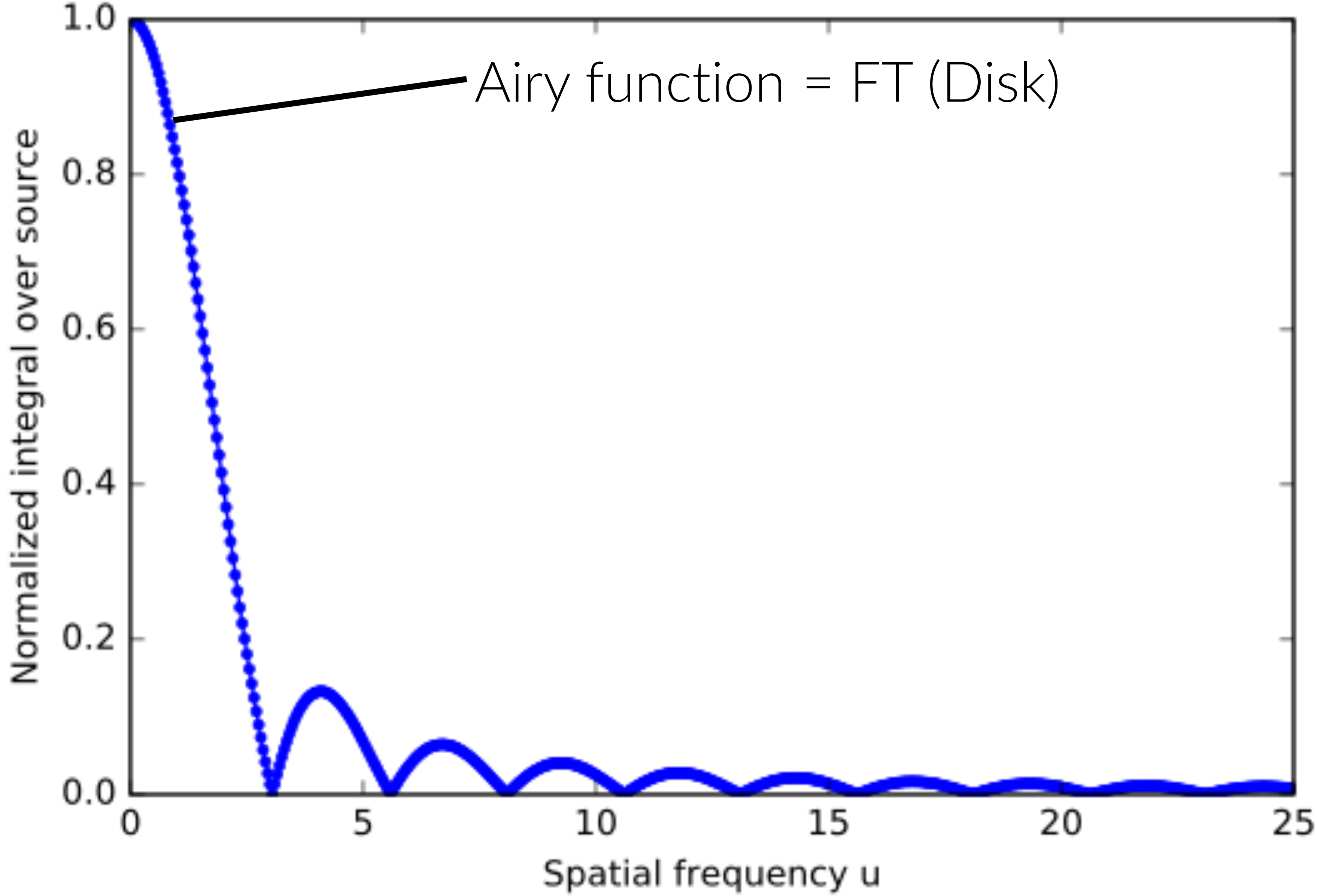

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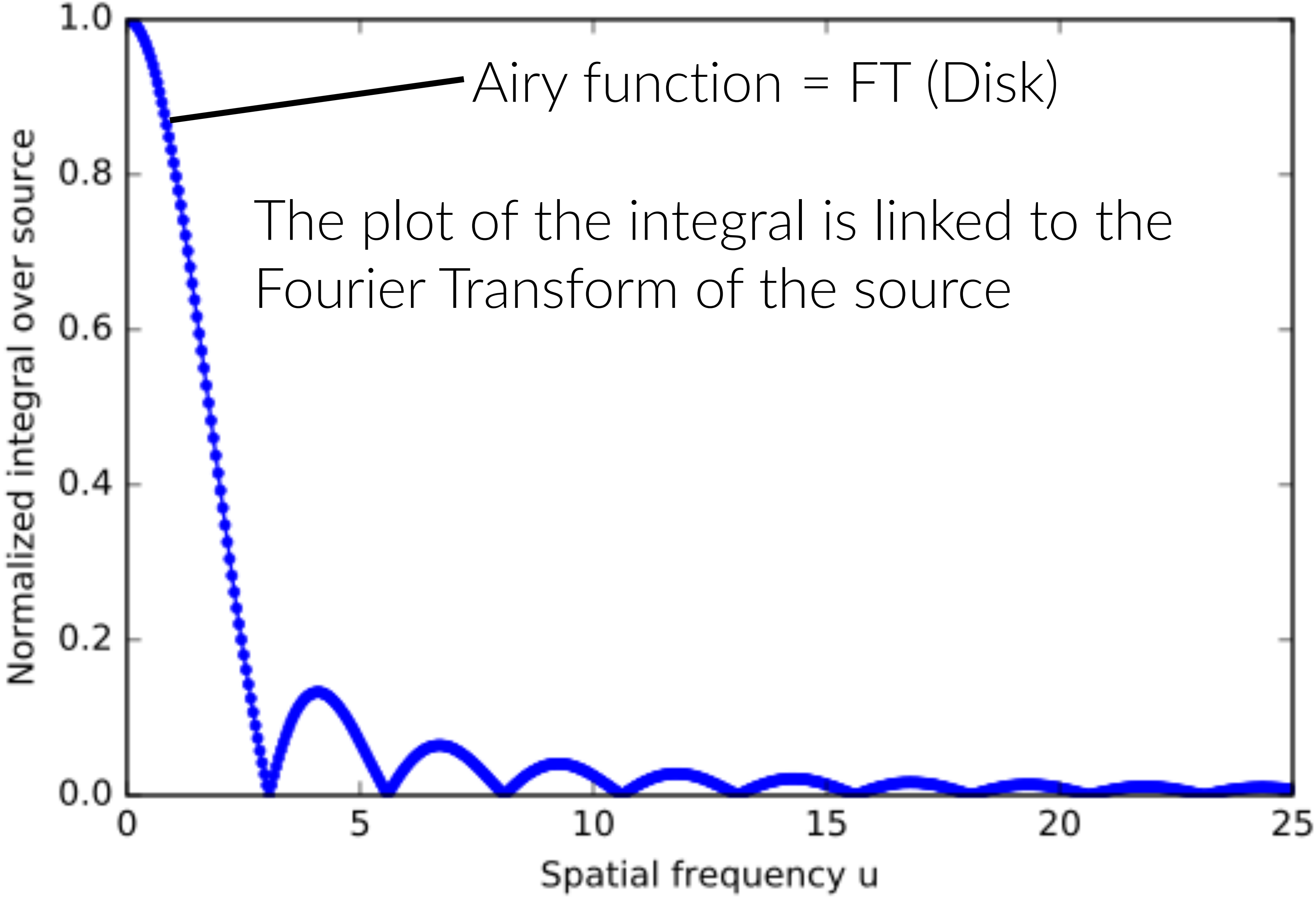

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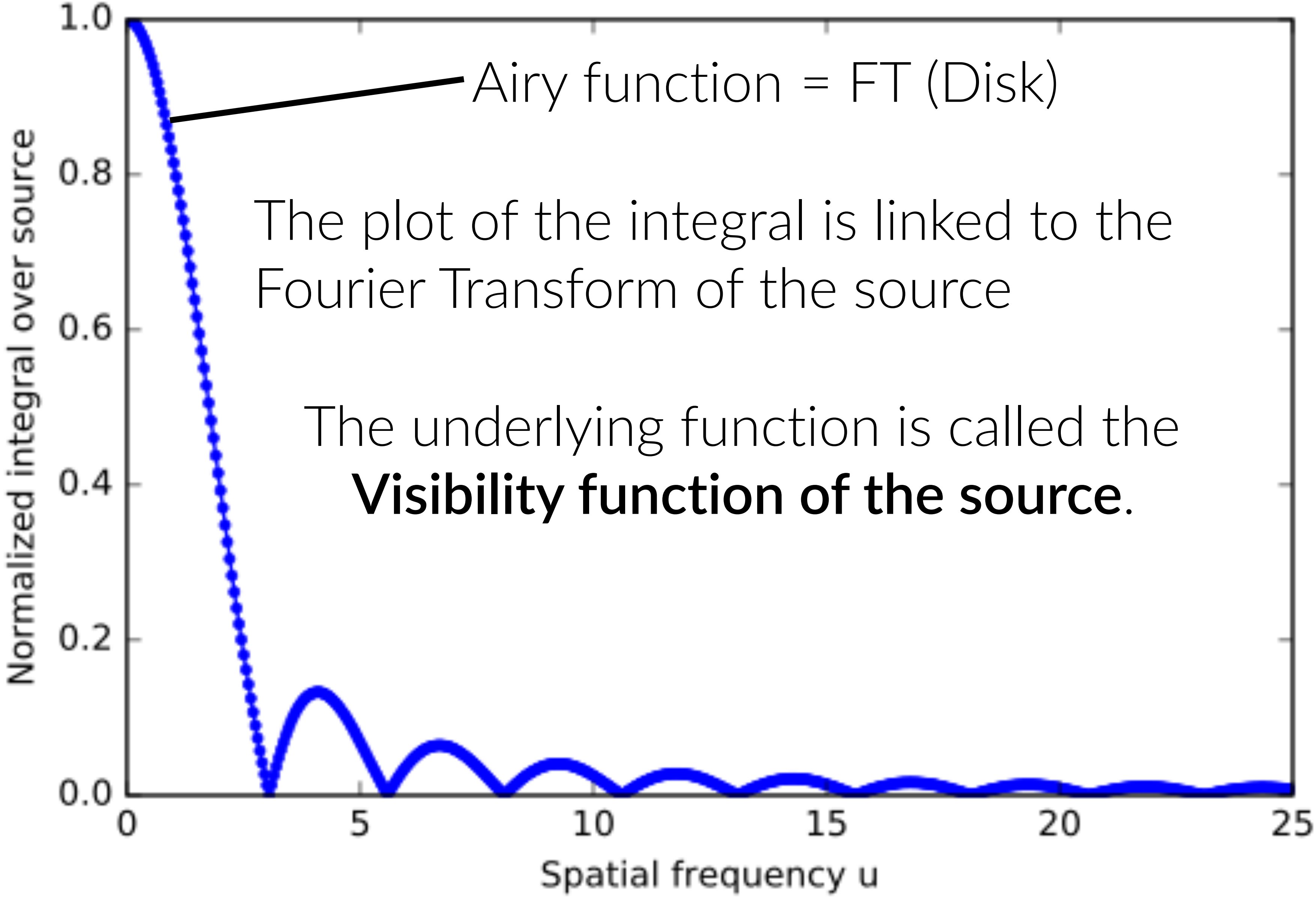

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The integral describes a function which is the **visibility function of the source**

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In real life, we only have a limited number of baselines, so that the **visibility function is not fully sampled**.

The lack of samples will have important consequences on the imaging capability with the interferometer

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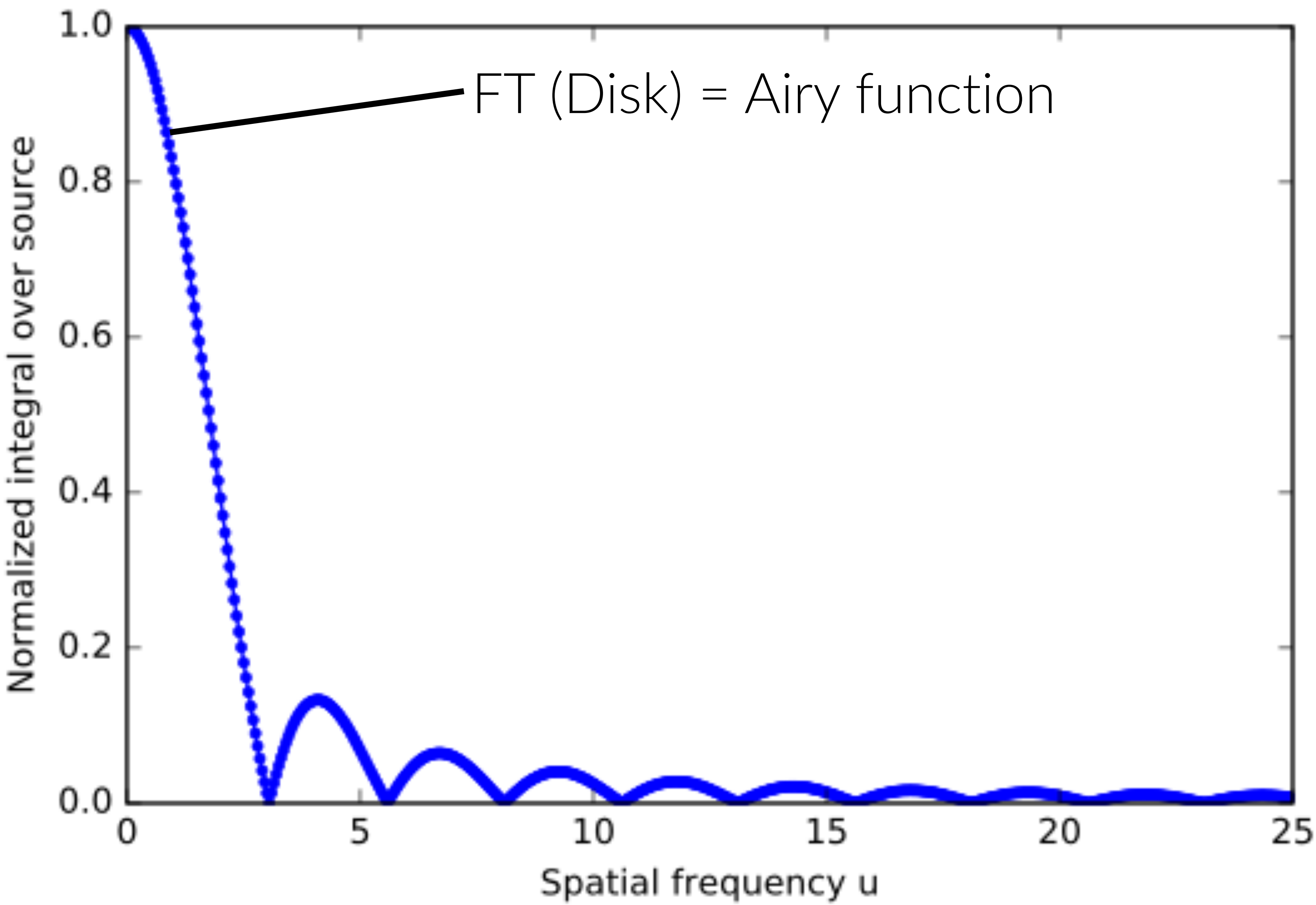
Let's compute the integral for a lot of different values of u



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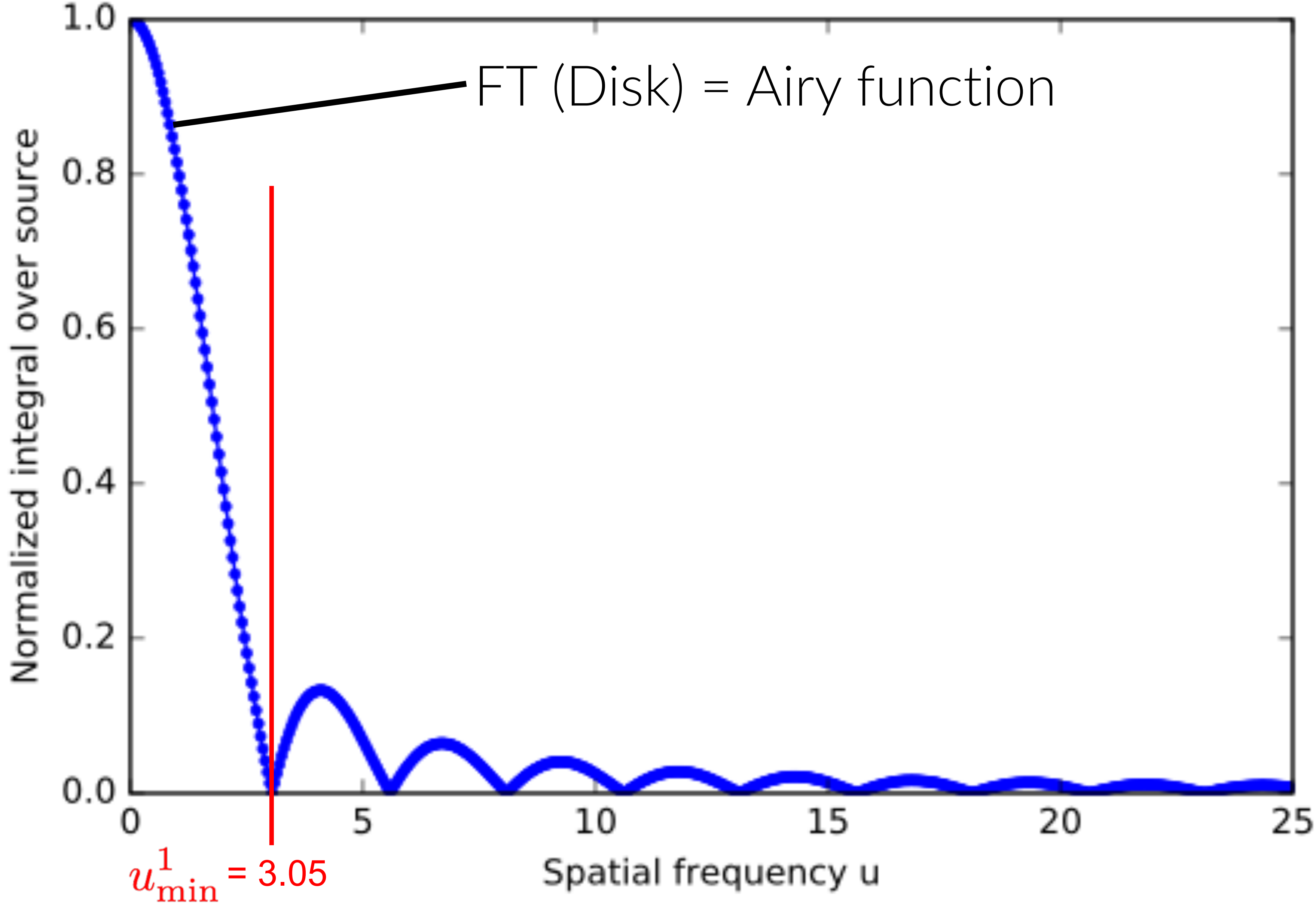

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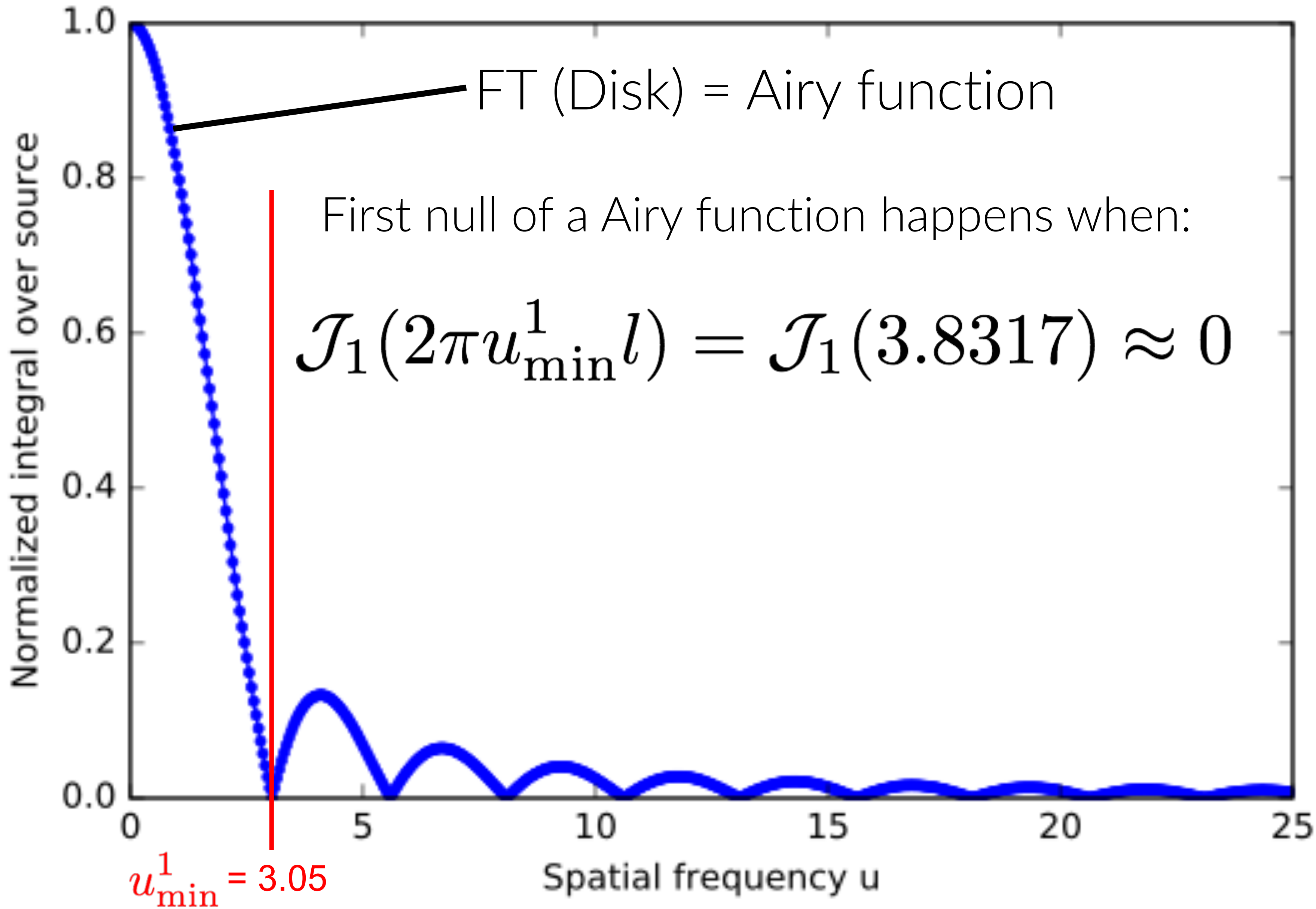

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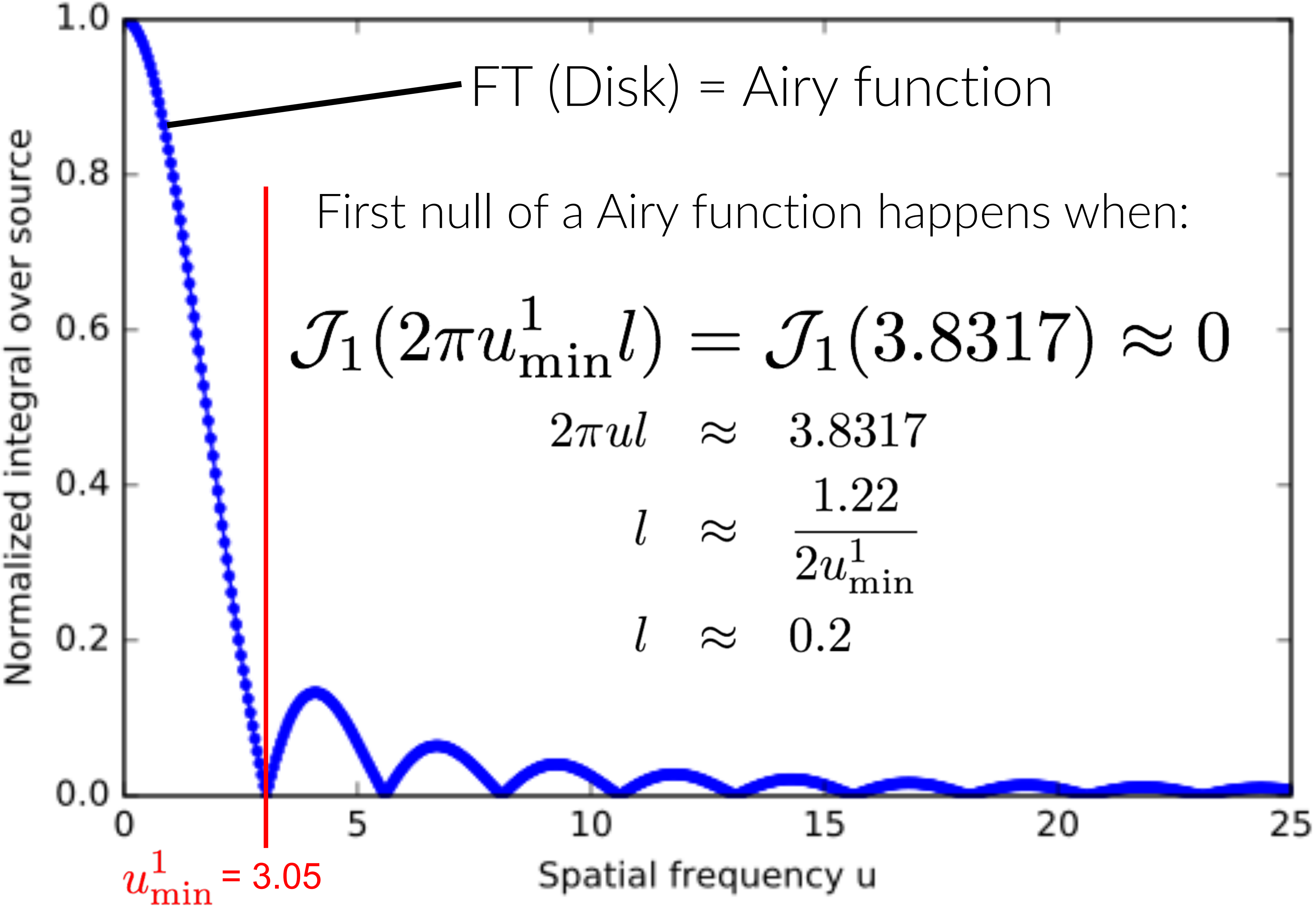

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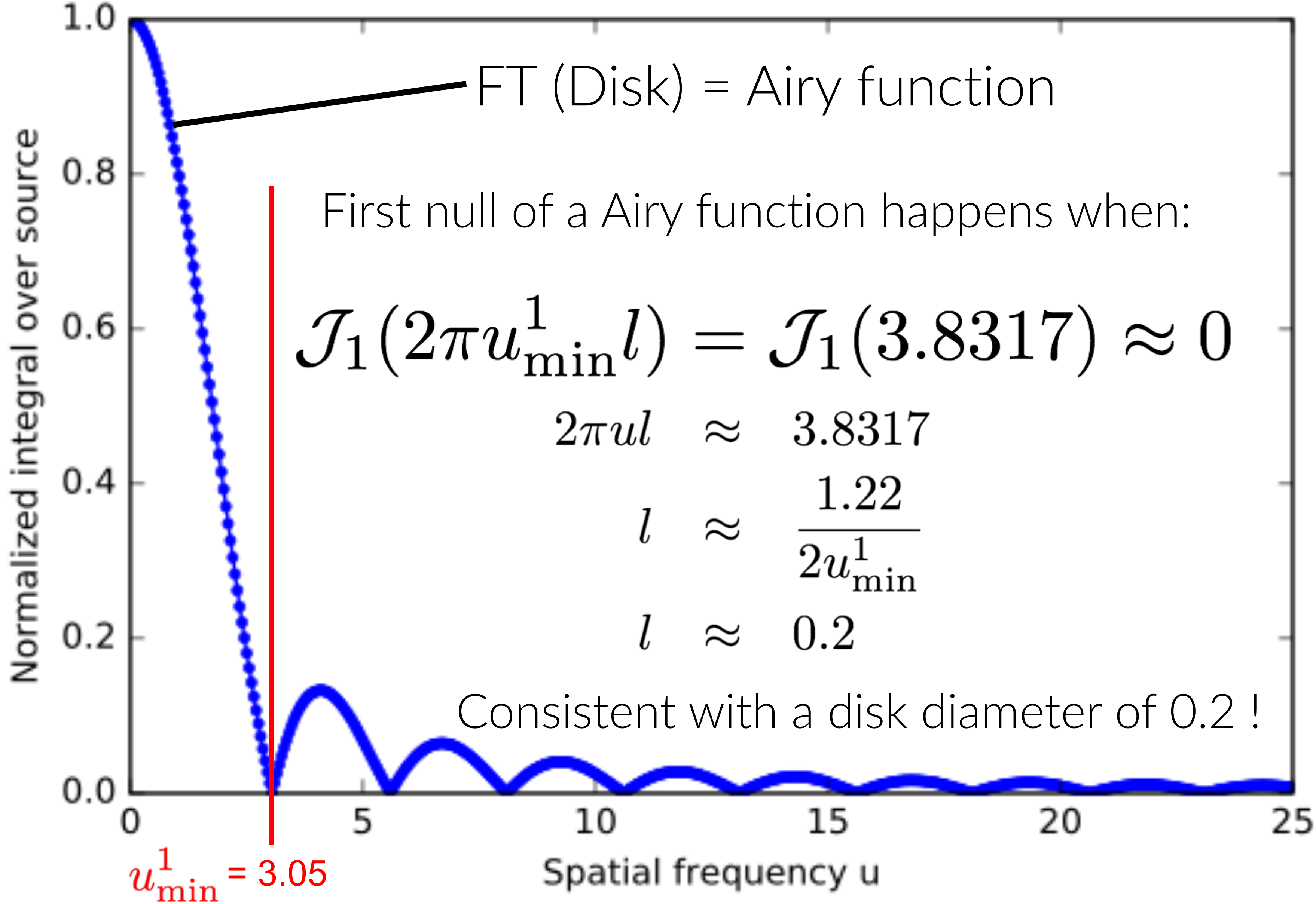

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In summary, an interferometer is a precise ruler which helps measuring the location and the size of object in the sky, even if they are not seen by a single receiver.

The need for complex correlation (complex visibility)

The 2-element interferometer : The Π interferometer

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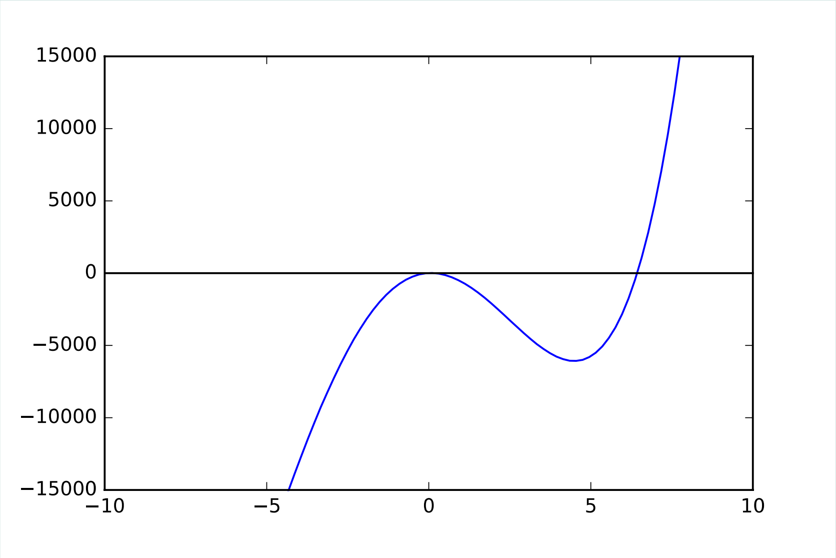
Any function can be decomposed into a even part and a odd part.



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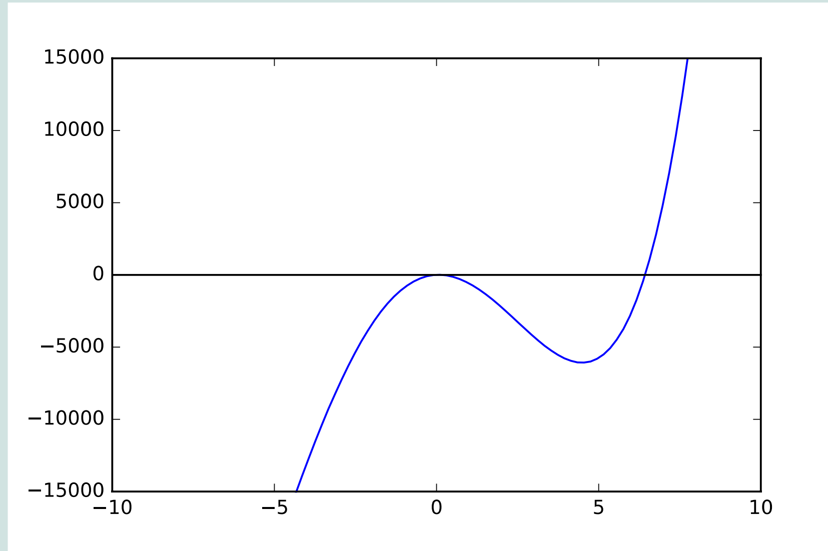
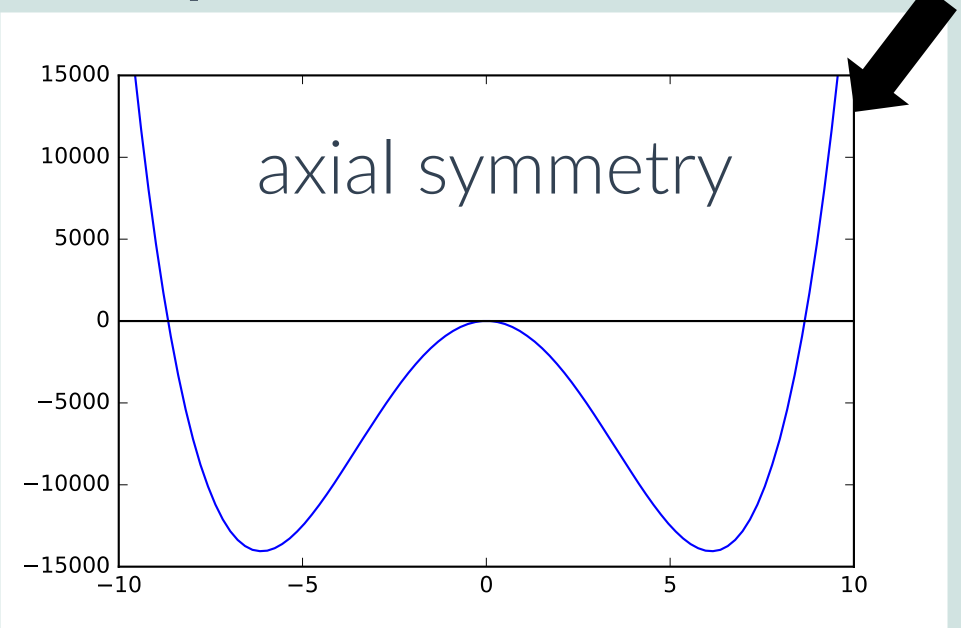
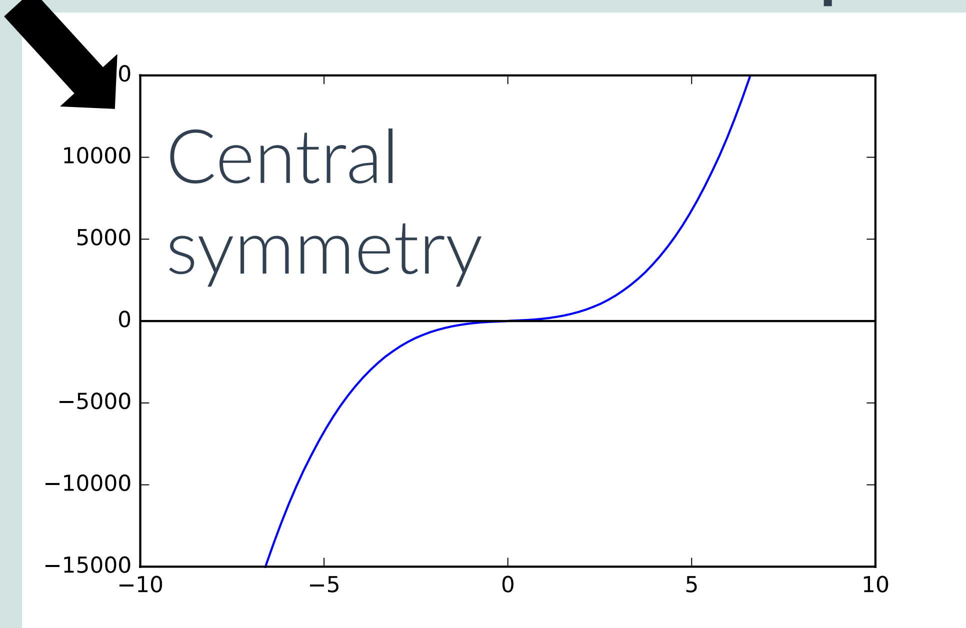


Even part



+

Odd part

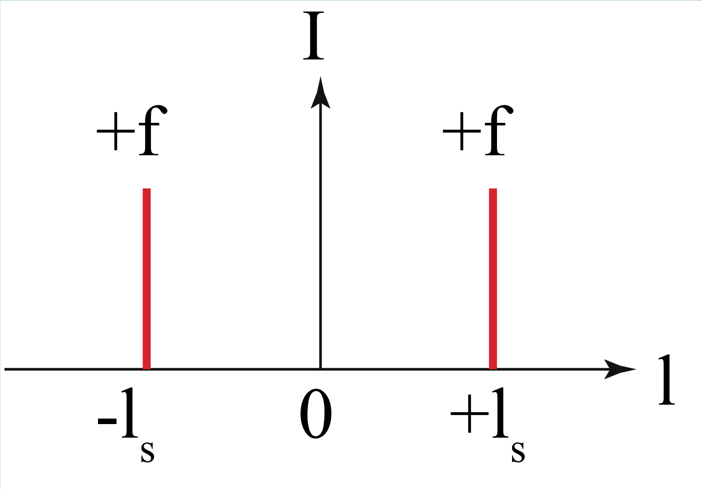


Problem !

A cosine correlator is only sensitive to the even part of the sky !

The 2-element interferometer : sensitivity to the sky

Let's consider a fake "even" sky



The sky intensity distribution can be separated in a odd and a even part

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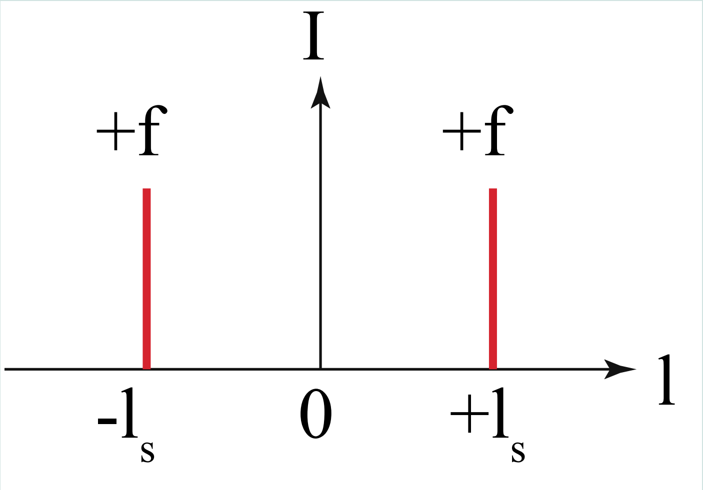

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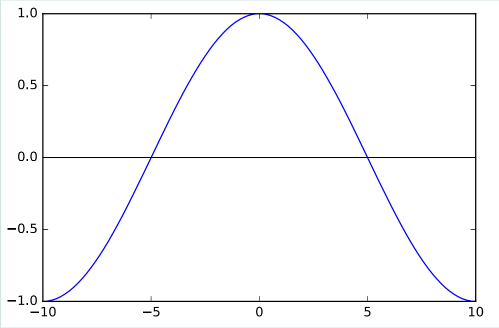

cosine correlator

$\int$

$\neq 0$

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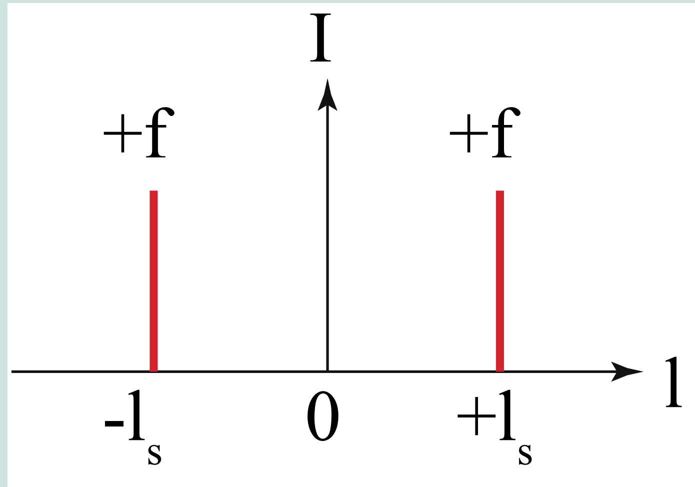

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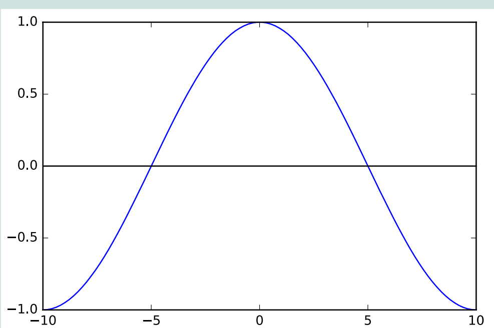
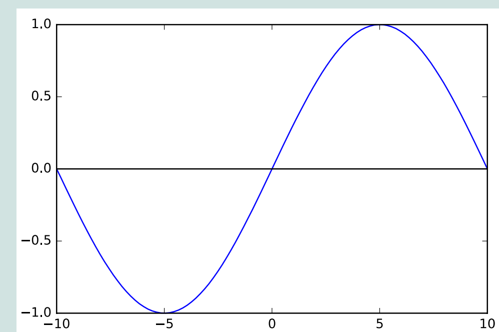


cosine correlator

sine correlator



$\int$

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$\int$

$= 0$

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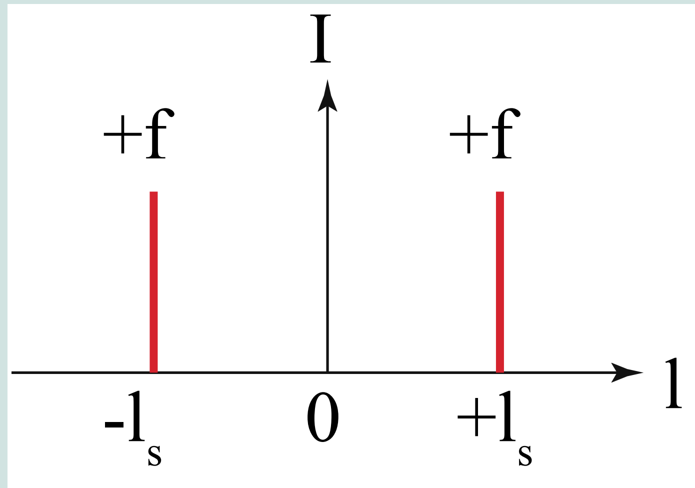

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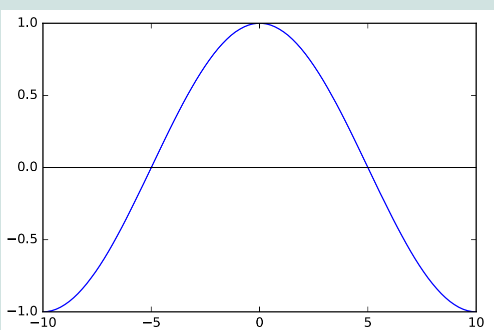

cosine correlator

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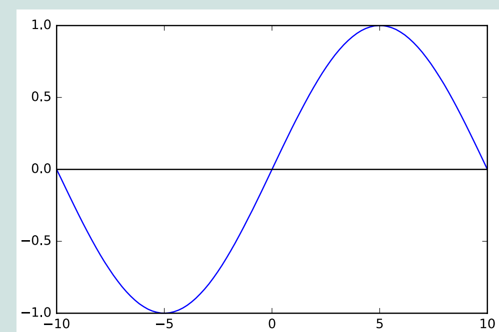

$\int$ $\int$

a cosine correlator is only sensitive to the even part of the sky

$\neq 0$ $= 0$

The 2-element interferometer : sensitivity to the sky

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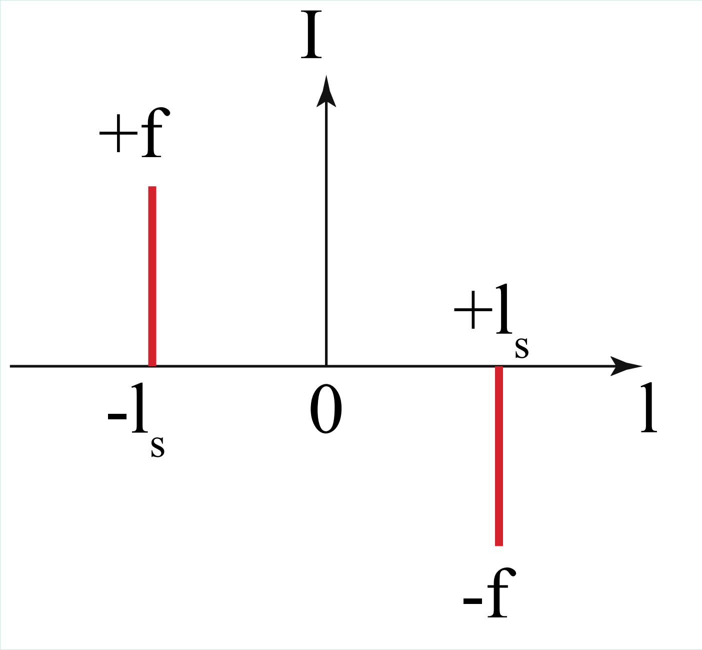

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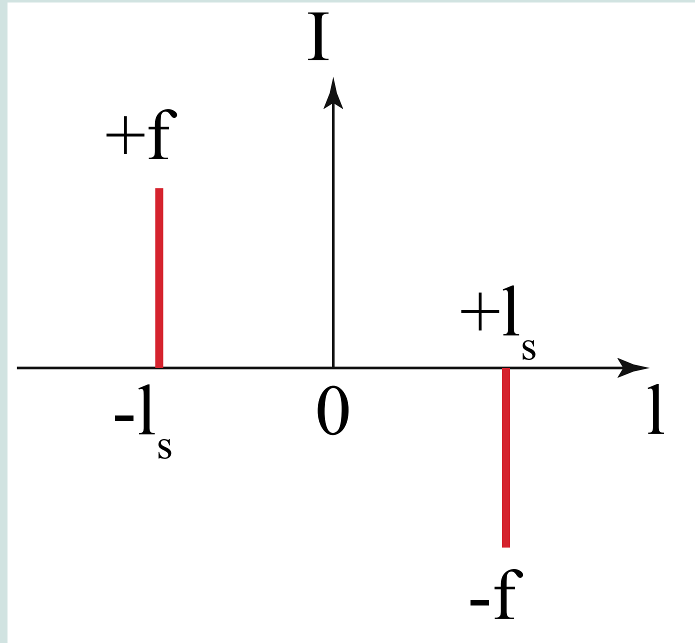

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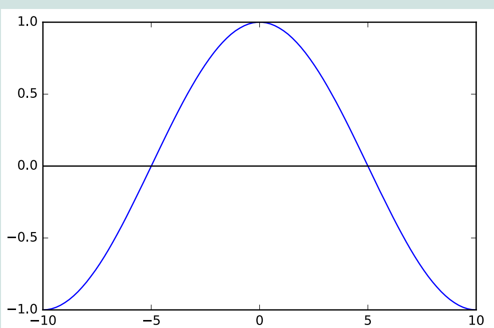

cosine correlator

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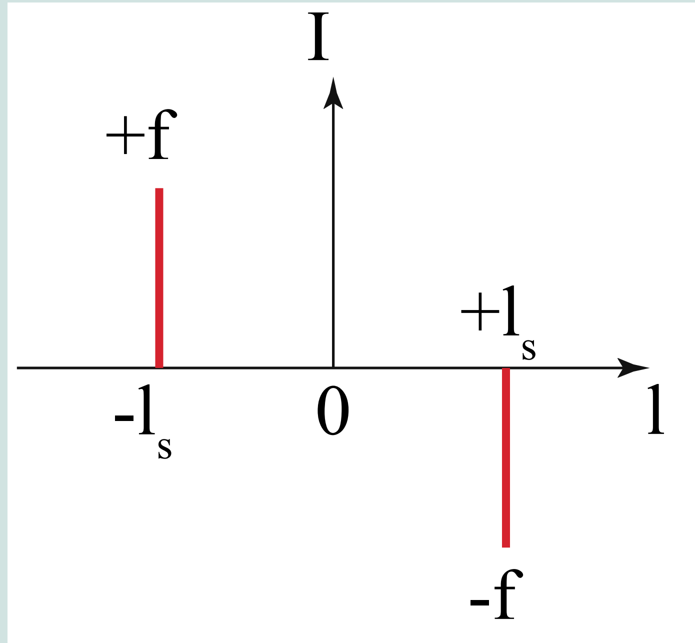

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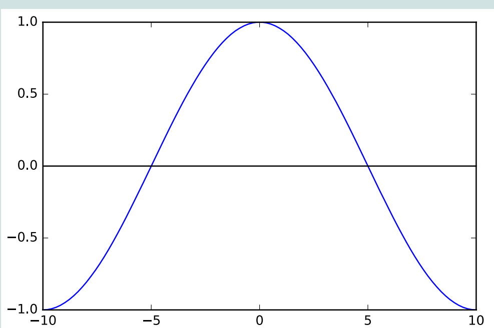
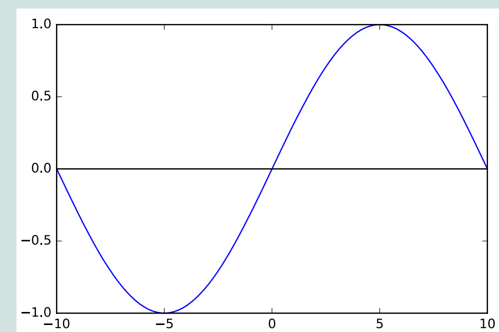


cosine correlator

$\int$

$= 0$



sine correlator

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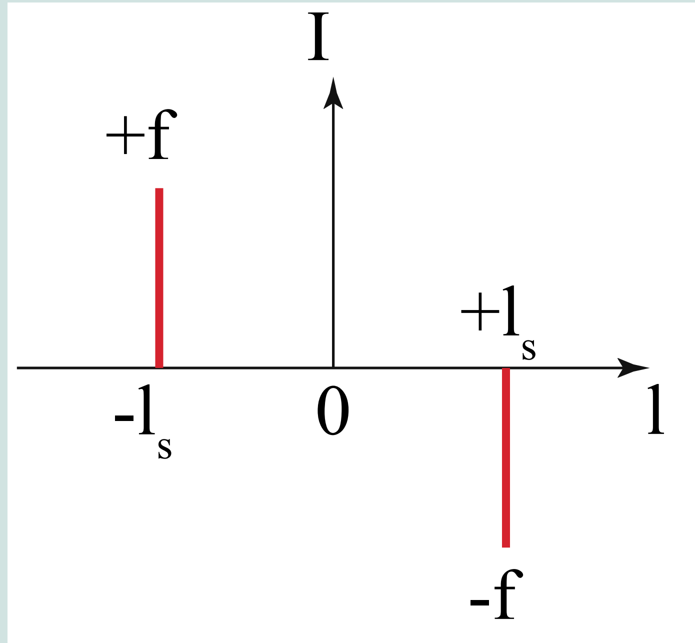

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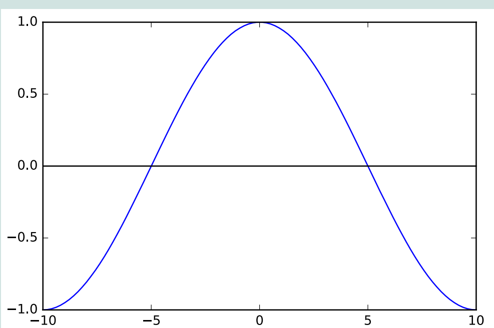

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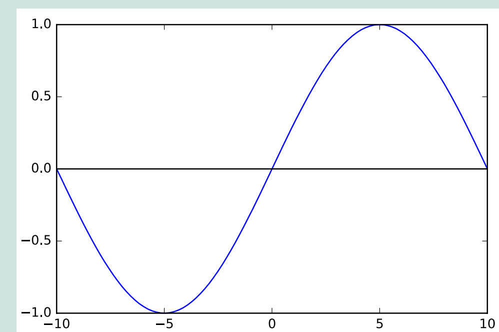
$= 0$

$*$



sine correlator

$\int$ ↓

$\neq 0$

a sine correlator is only sensitive to the odd part of the sky

The 2-element interferometer : sensitivity to the sky

To solve this problem, we will build another correlator using the "sine" function

In practice, we need to shift one of the two signals by an extra phase of $\frac{\pi}{2}$, inserting a sine function in the correlation:

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$$\underbrace{\langle V_{01} V_{02} \sin \omega t \cos [\omega (t + \tau)] \rangle_t}$$

with $\tau = \frac{\Delta L}{c}$

$$C = V_0^2 \frac{\langle \sin(2\omega t + \tau) + \sin(\omega \tau) \rangle_t}{2}$$

The 2-element interferometer : The $\prod$ interferometer

Again, once applying a time averaging, the temporal oscillations caused by

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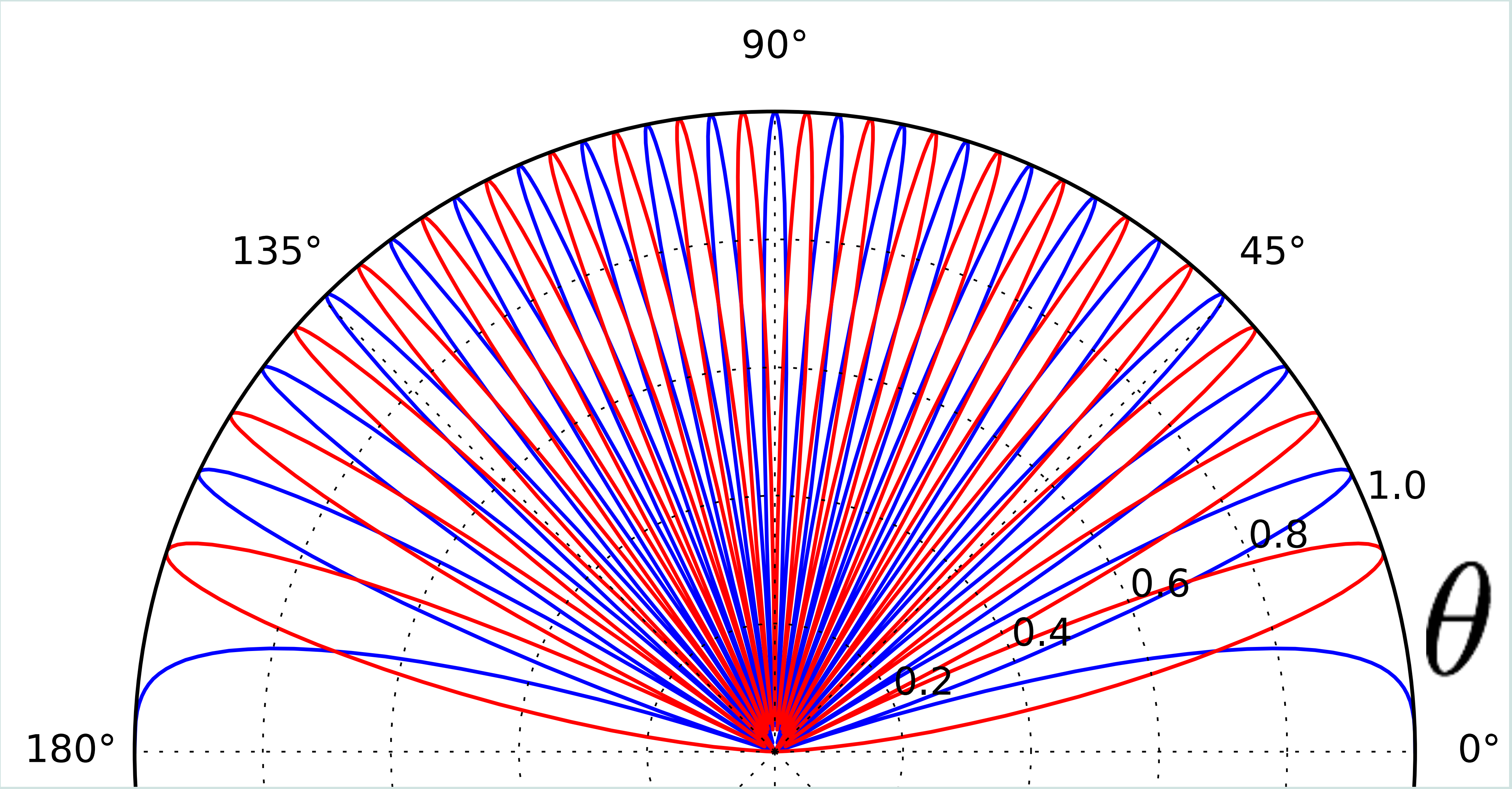
It is equivalent to filter the signals with a low-pass filter which role is to remove the fast-varying component of the signal.

$$C_{\sin} = \frac{V_0^2}{2} \sin \omega\tau$$

The 2-element interferometer : The complex correlator

On this plot are represented the fringe pattern projected on the sky, associated with a **cosine correlator** (blue) and a **sine correlator** (red)



The 2-element interferometer : The complex correlator

To be able to measure the whole distribution of intensity of the sky, we need to combine the measurements of the odd part and the even part of it.

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Nowadays, the cross-correlation from the two antenna signals is made directly in complex form mimicking the former use of a sine et cosine correlator.

IBM BlueGene/P used for LOFAR correlation

The 2-element interferometer : The $\prod$ interferometer

$$C_{\cos} = \frac{V_0^2}{2} \cos \omega\tau \qquad C_{\sin} = \frac{V_0^2}{2} \sin \omega\tau$$

The 2-element interferometer : The Π interferometer

$$C_{\cos} = \frac{V_0^2}{2}\cos\omega\tau \qquad C_{\sin} = \frac{V_0^2}{2}\sin\omega\tau$$

$$\underline{C} = \Re(\underline{C}) - \imath\Im(\underline{C}) = C_{\cos} - \imath C_{\sin}$$

$$\underline{C} = \sum_{n=0}^{\infty} I_n \cos(2\pi\frac{\Delta L_n}{\lambda}) - \imath\sum_{n=0}^{\infty} I_n \sin(2\pi\frac{\Delta L_n}{\lambda})$$

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$$\underline{C} = \Delta\nu\int_{\Omega} A(\mathbf{s}) I_\nu(\mathbf{s}) e^{-\imath 2\pi\frac{\mathbf{b}\cdot\mathbf{s}}{\lambda}}\, d\Omega$$

The 2-element interferometer : multiple directions

In our 1D case, we will explicit $\mathbf{b} \cdot \mathbf{s}$

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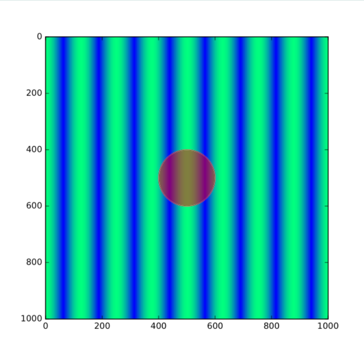
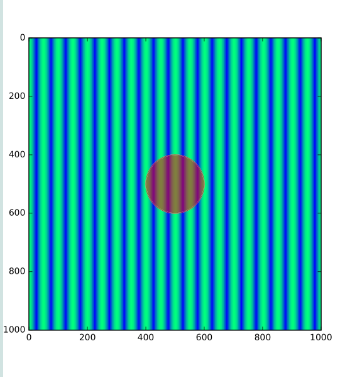
$$\underline{V} = |V| e^{\imath \phi_V} = \int_{\Omega} A(\boldsymbol{\sigma}) I_{\nu}(\boldsymbol{\sigma}) e^{-\imath 2\pi \frac{\mathbf{b}\cdot\boldsymbol{\sigma}}{\lambda}} d\Omega$$

The visibility function

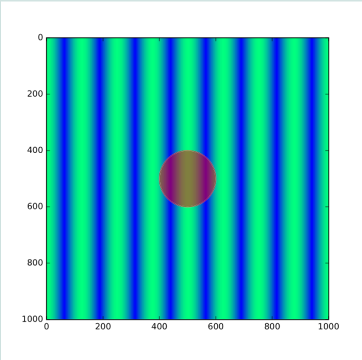
the UV plane

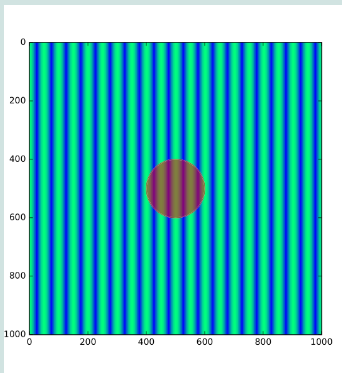
The 2-element interferometer : uv plane

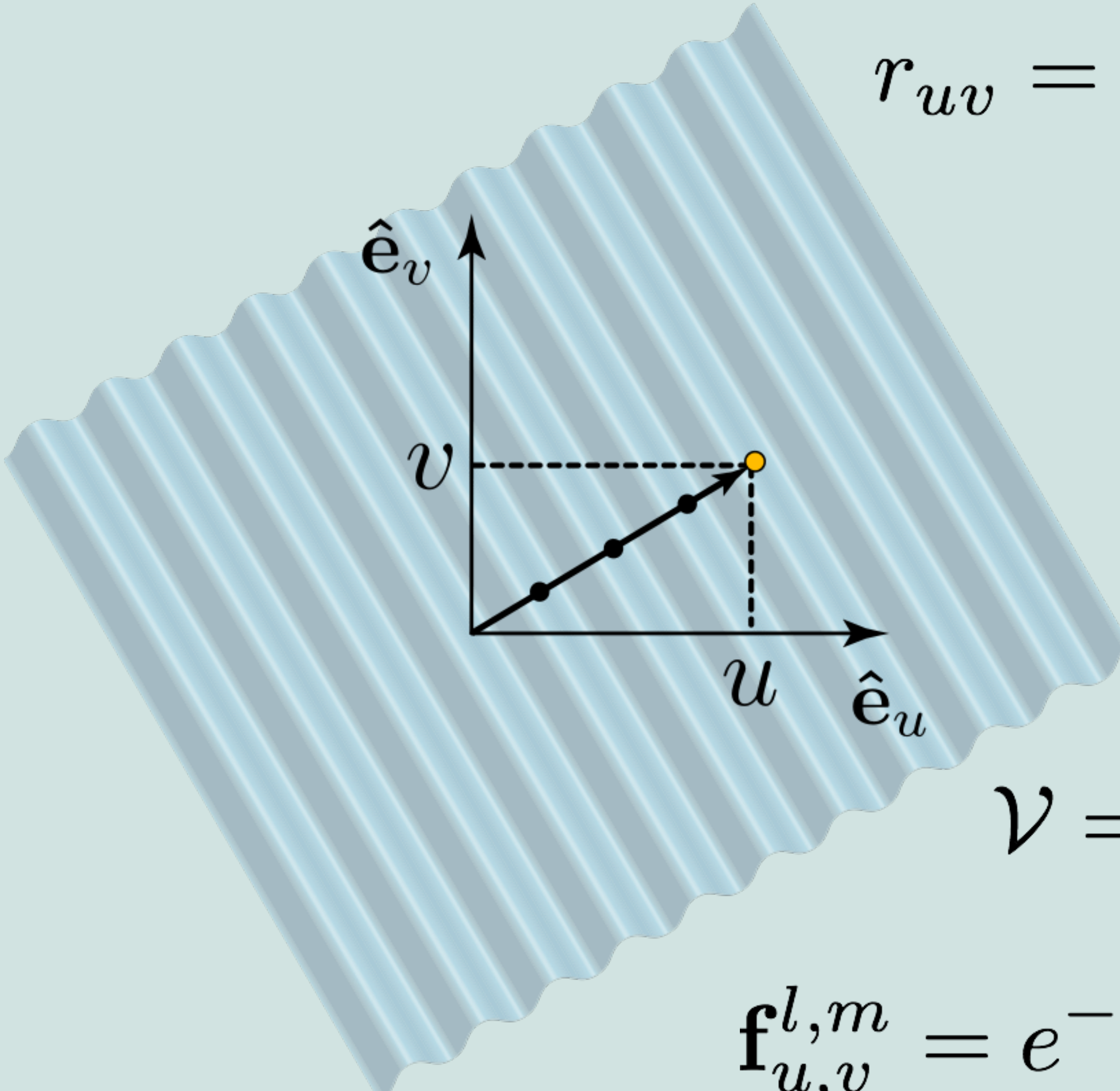




The 2-element interferometer : uv plane





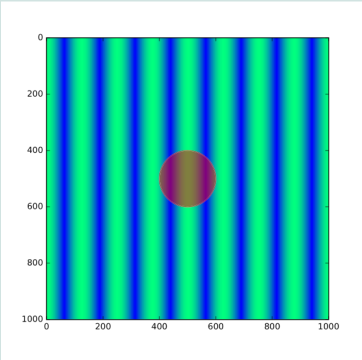


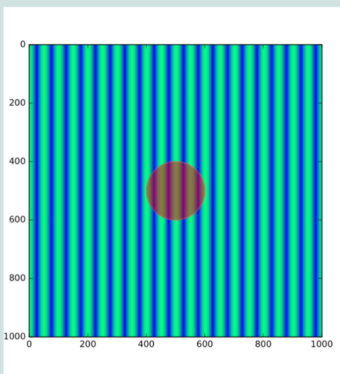
$$r_{uv} = \sqrt{u^2 + v^2}$$

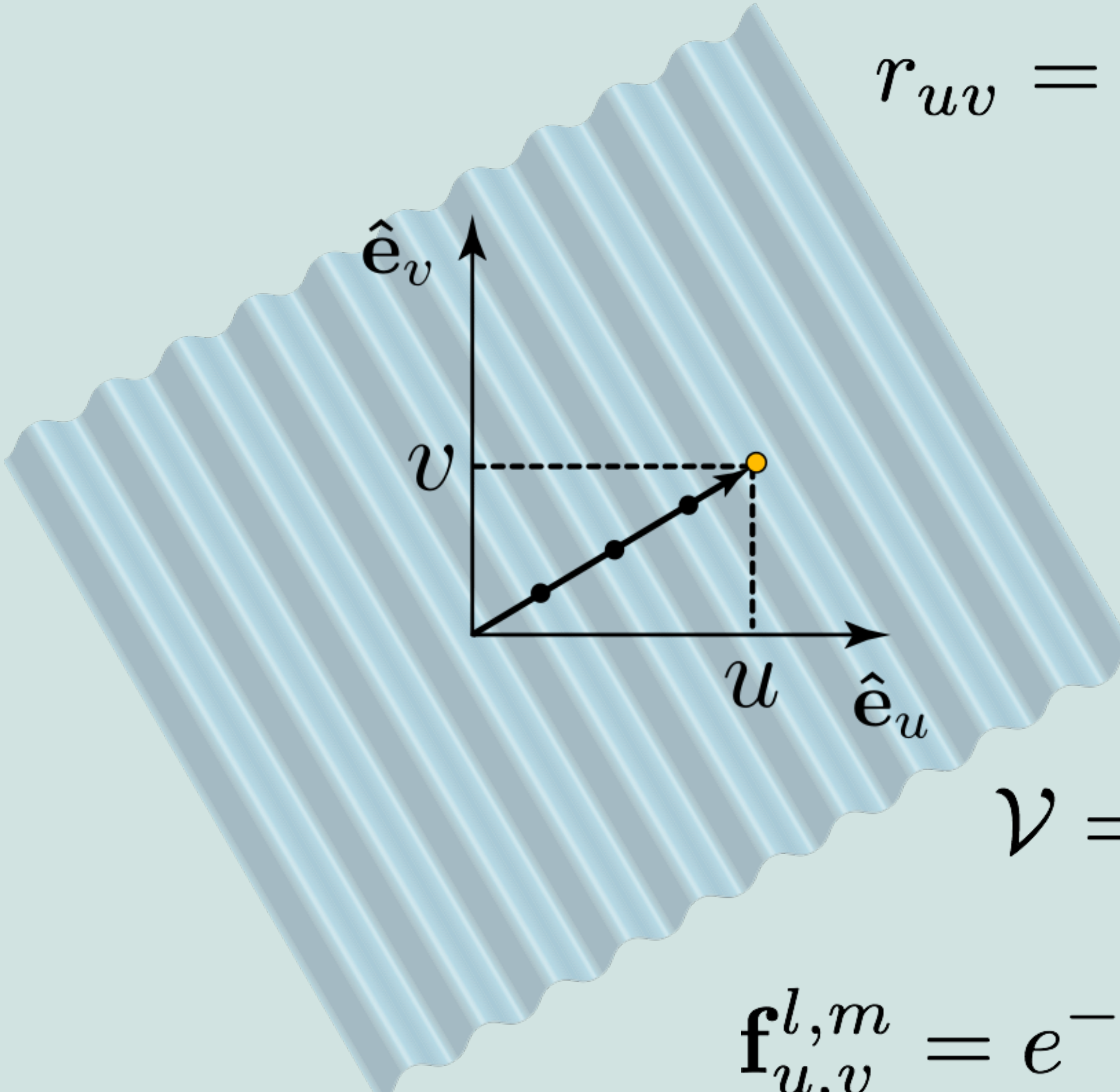
$$\mathcal{V} = \langle \mathbf{I}_\nu \cdot \mathbf{f}_{u,v}^{l,m} \rangle$$

$$\mathbf{f}_{u,v}^{l,m} = e^{-2j\pi(ul+vm)}$$

The 2-element interferometer : uv plane







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UV plane

2D plane containing the collection of all (u, v) measurements

The 2-element interferometer : uv plane

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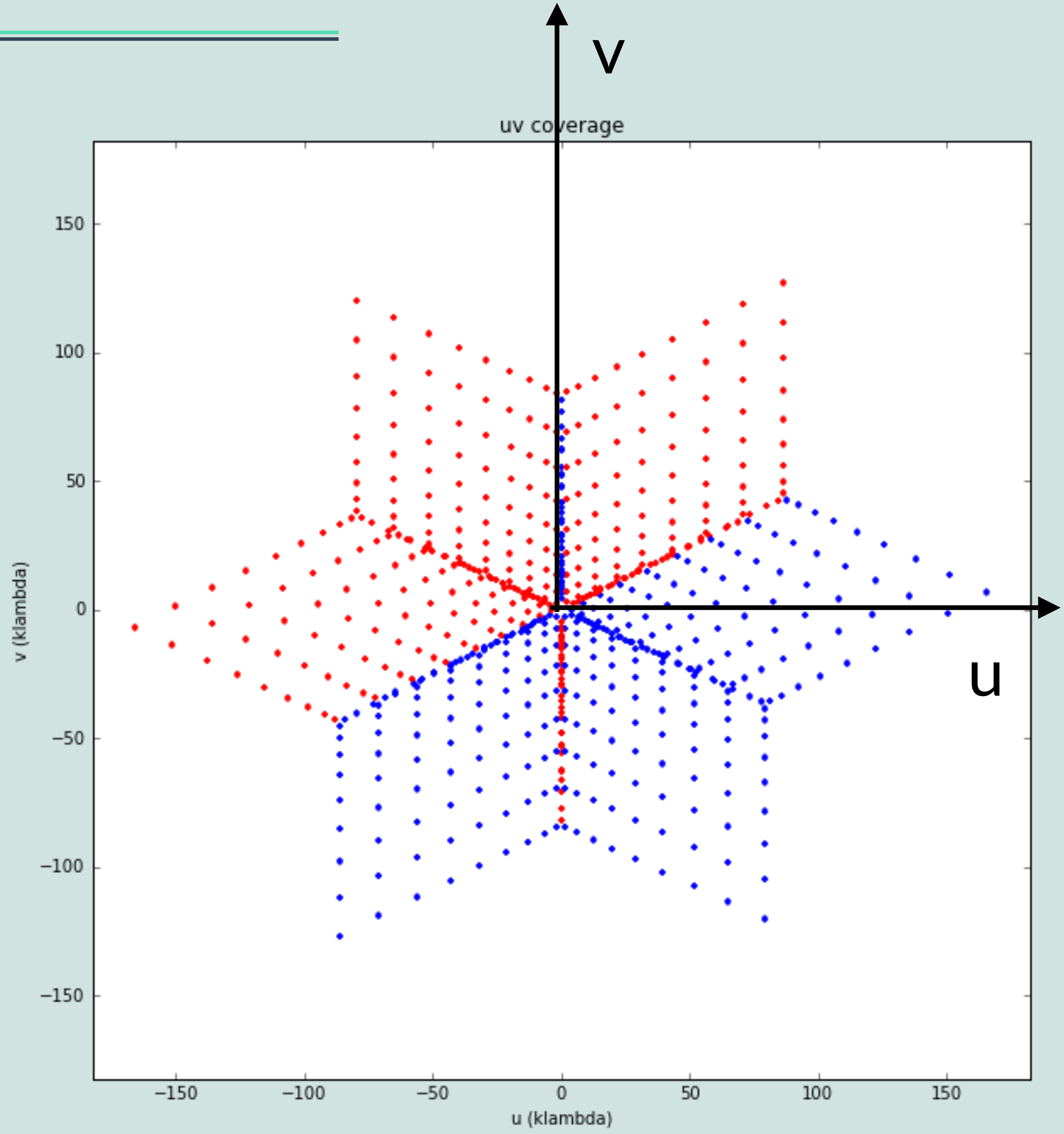
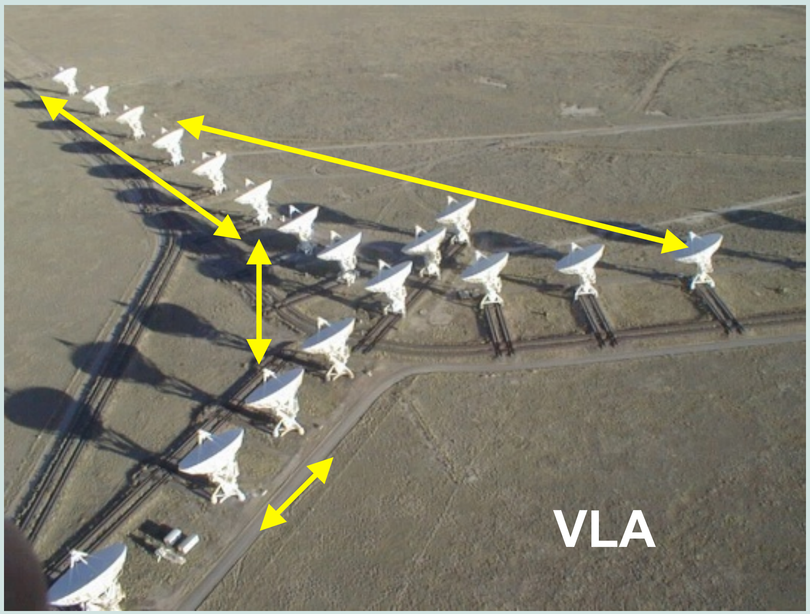
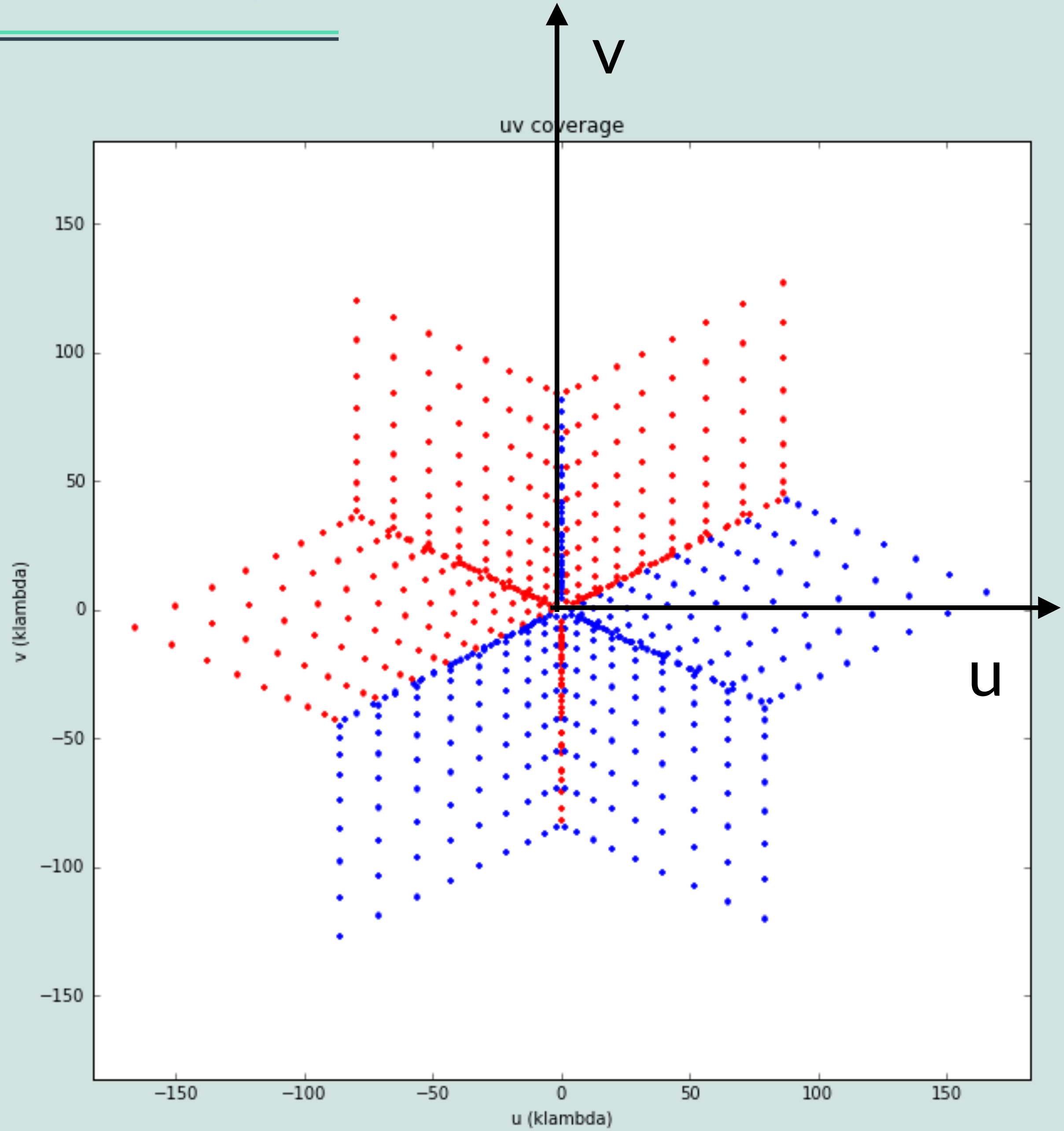
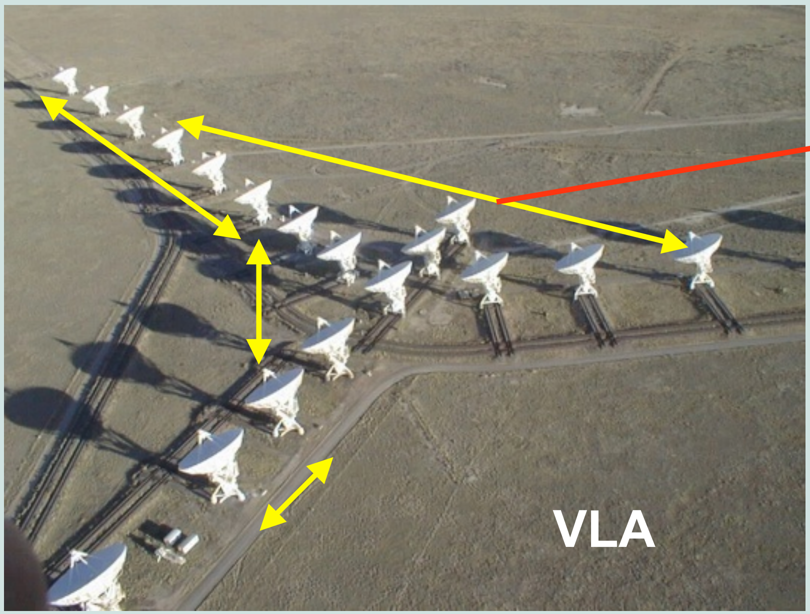


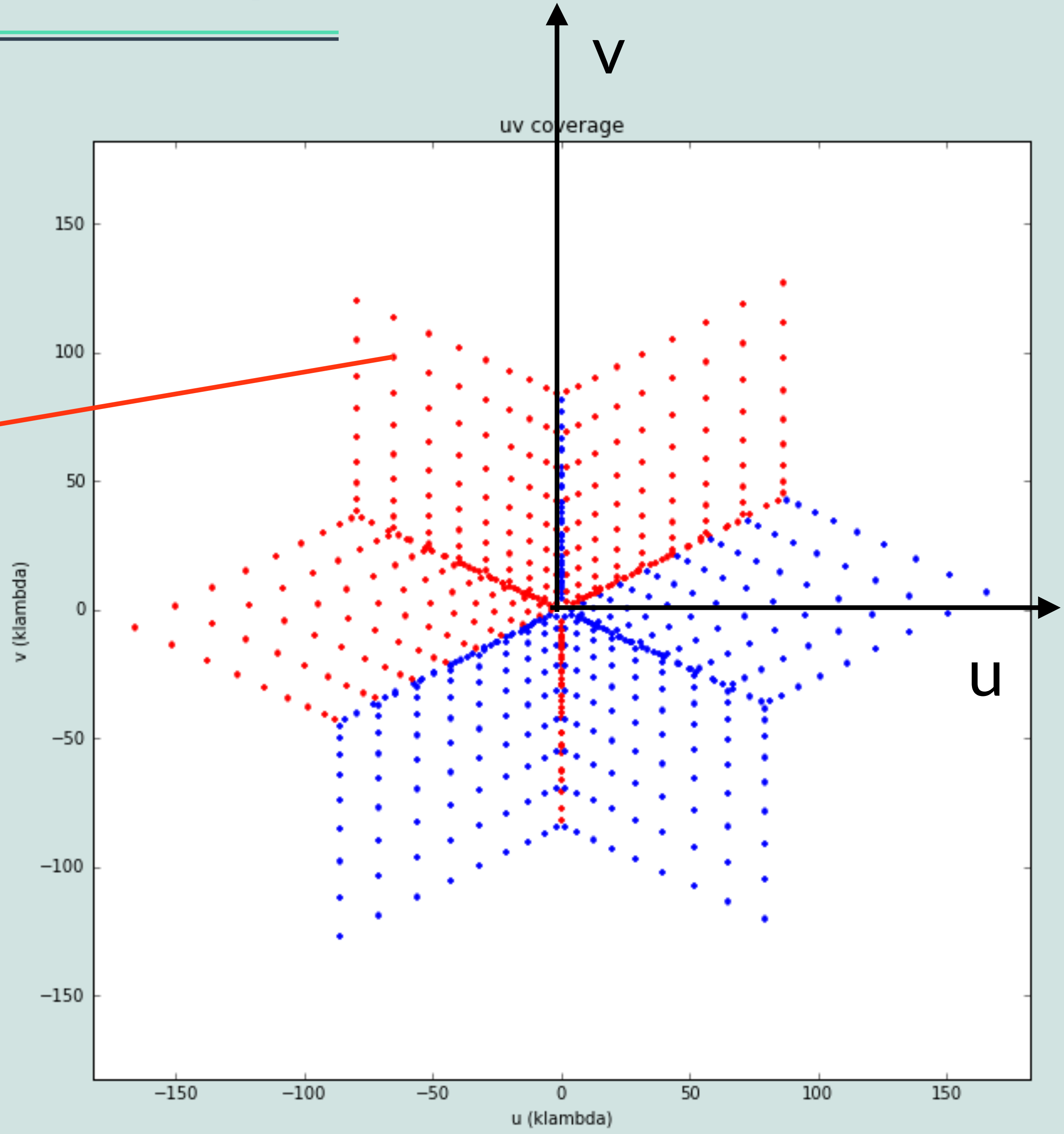
The 2-element interferometer : uv plane





The 2-element interferometer : uv plane





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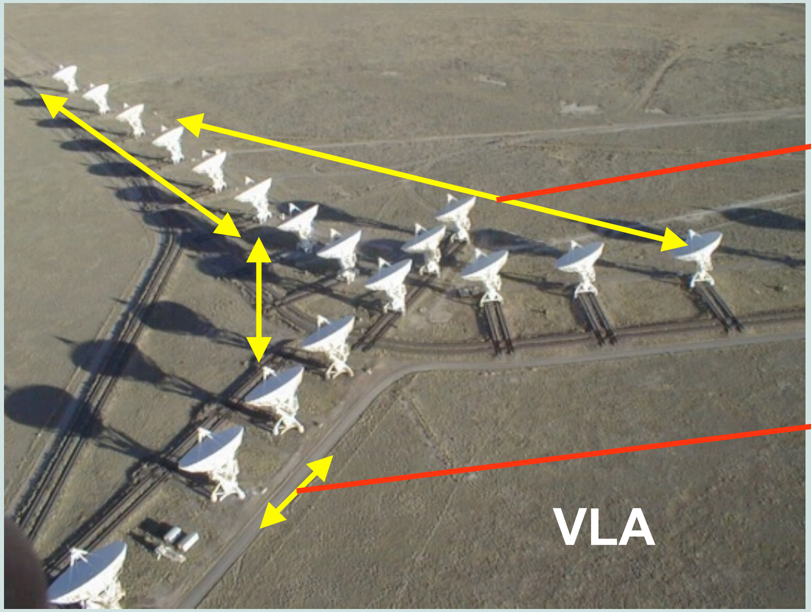
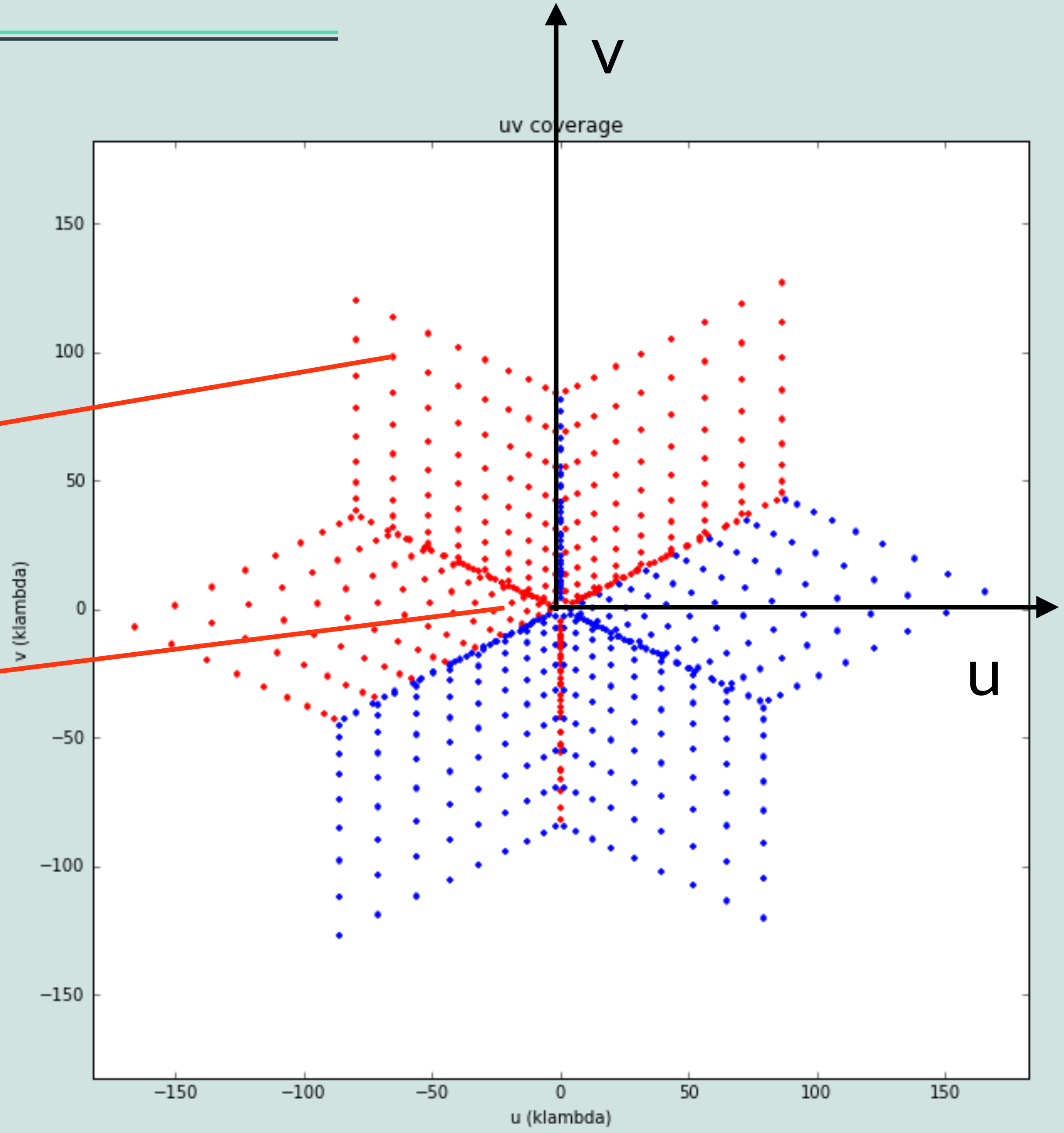


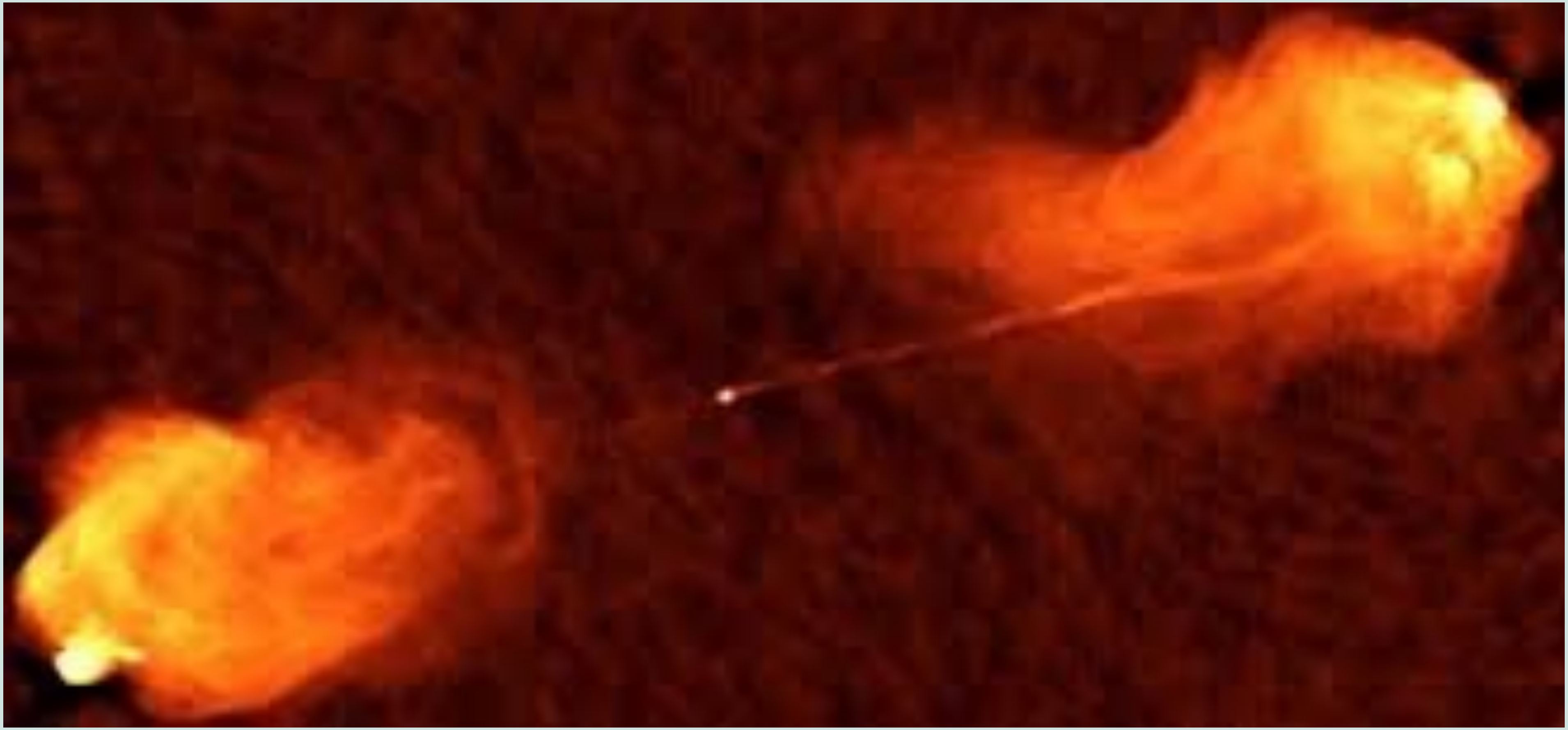


The 2-element interferometer : uv plane

Cygnus A

4.8 GHz

~2’ x ~1’



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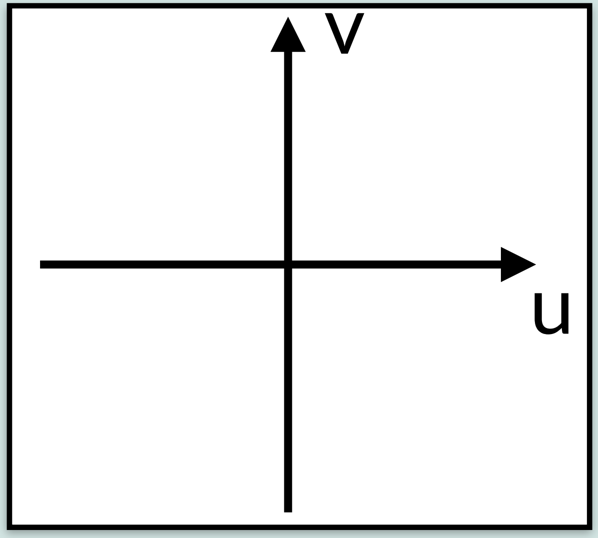

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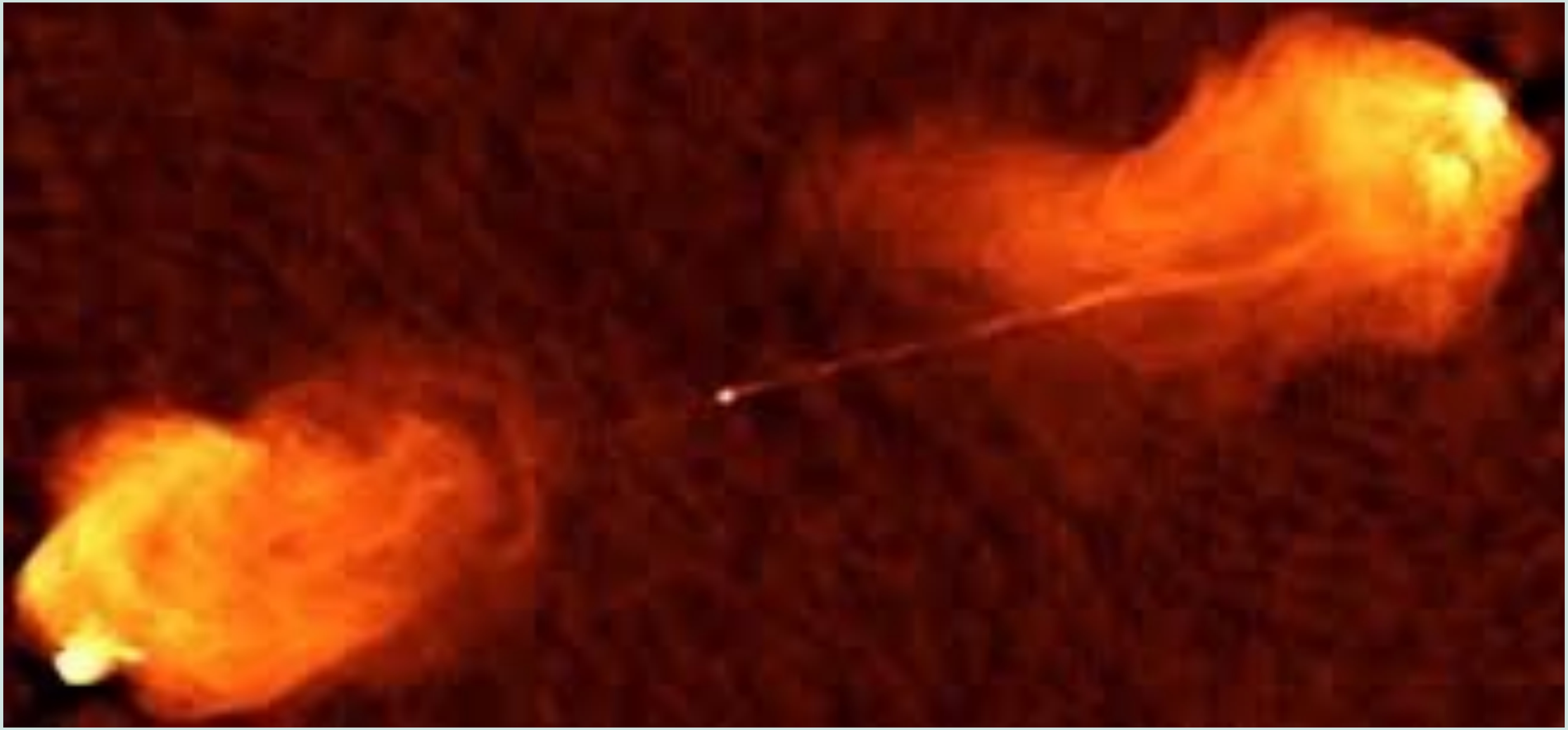

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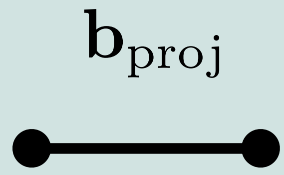

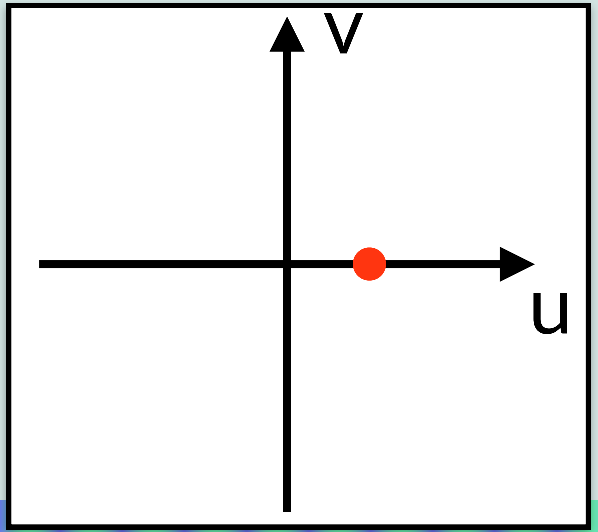

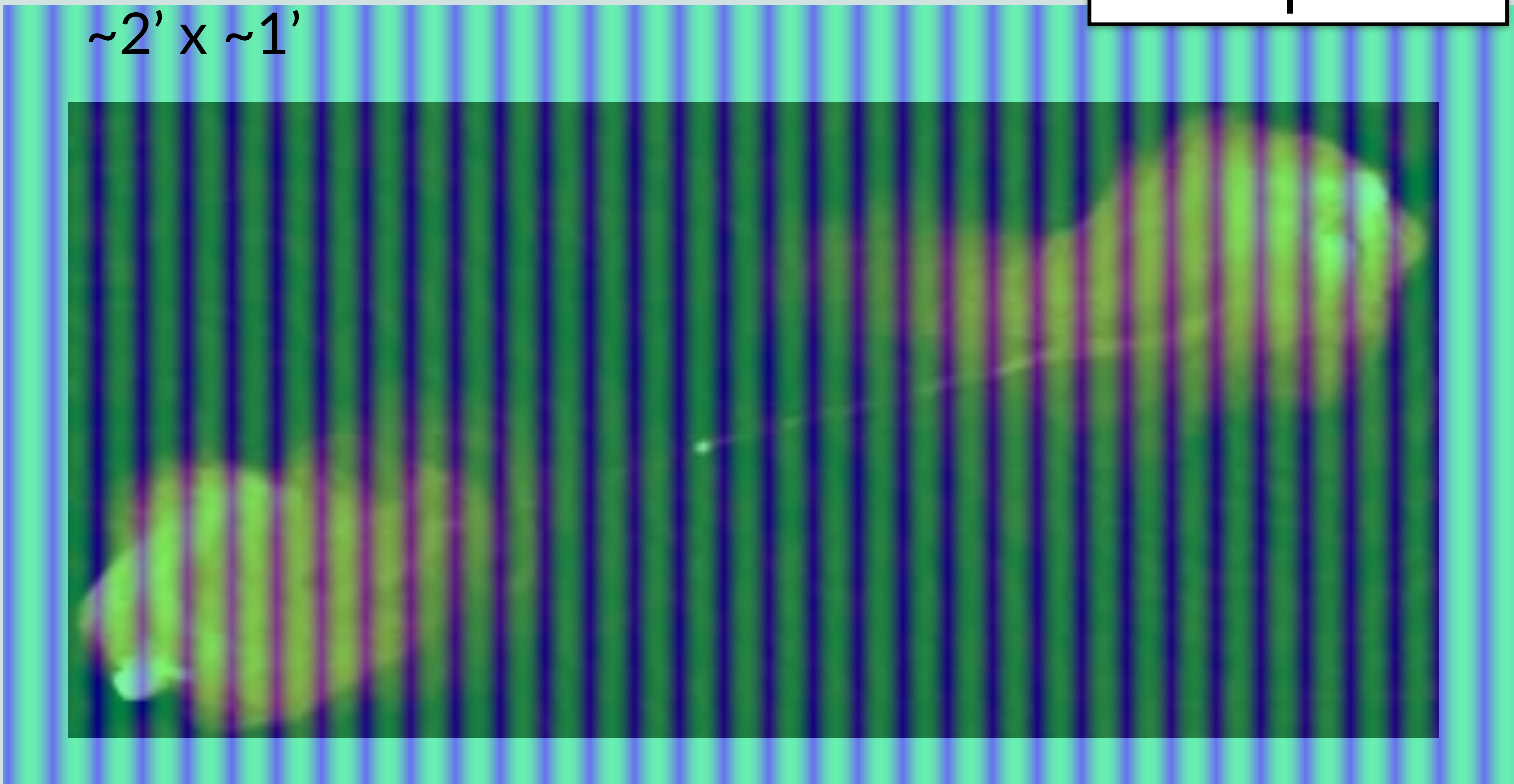

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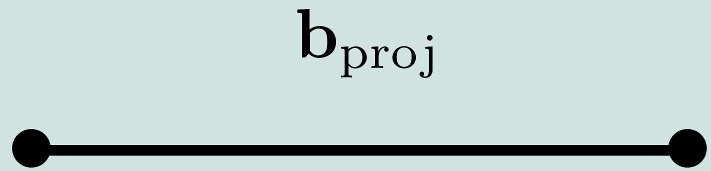

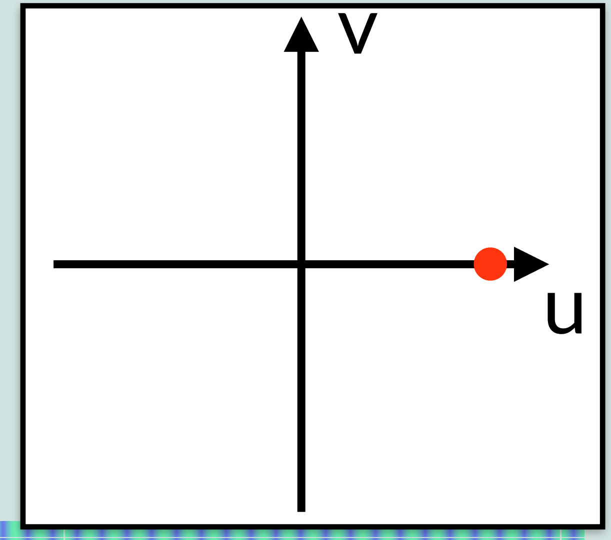

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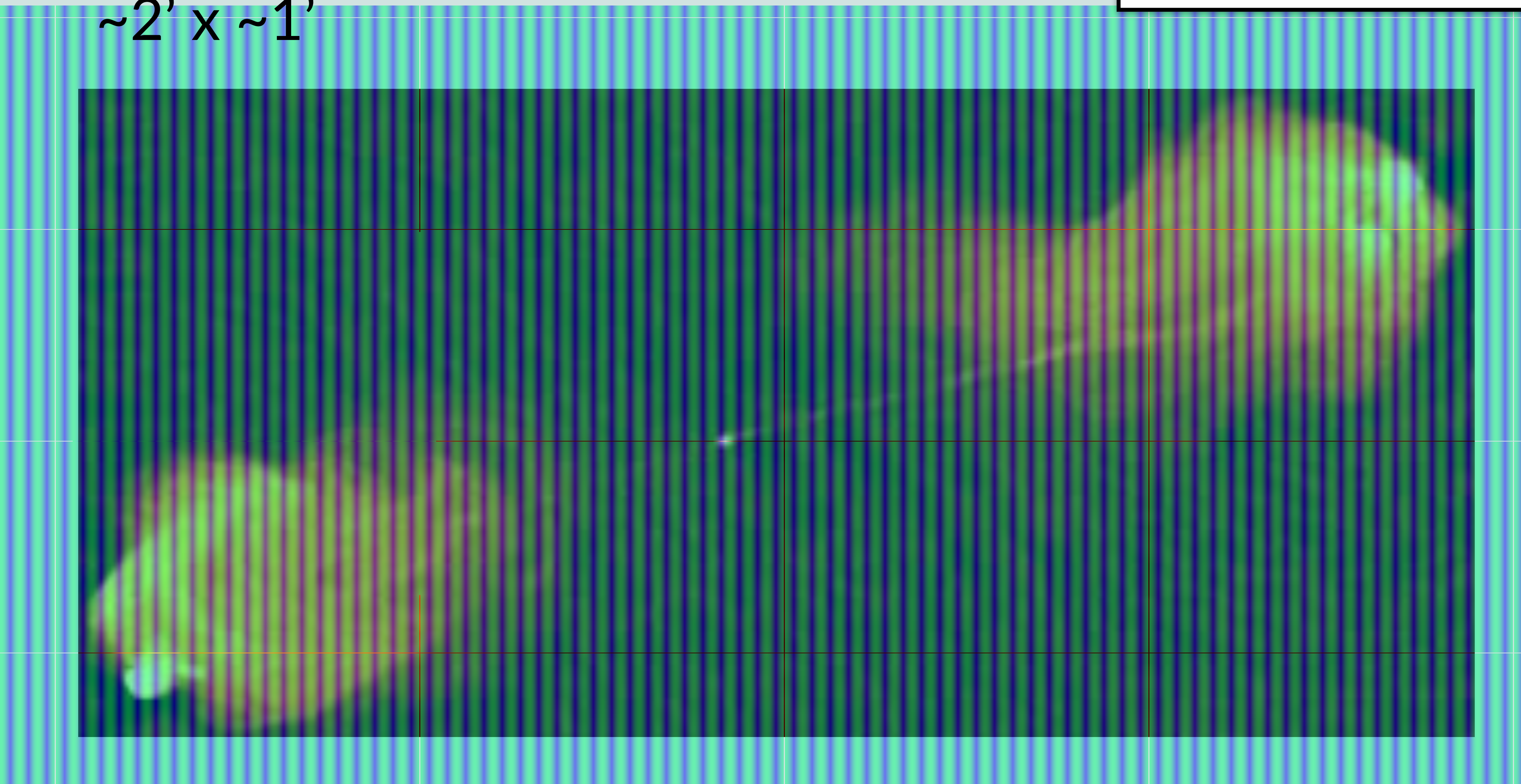


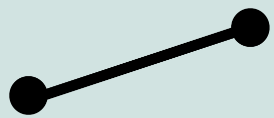
The 2-element interferometer : uv plane

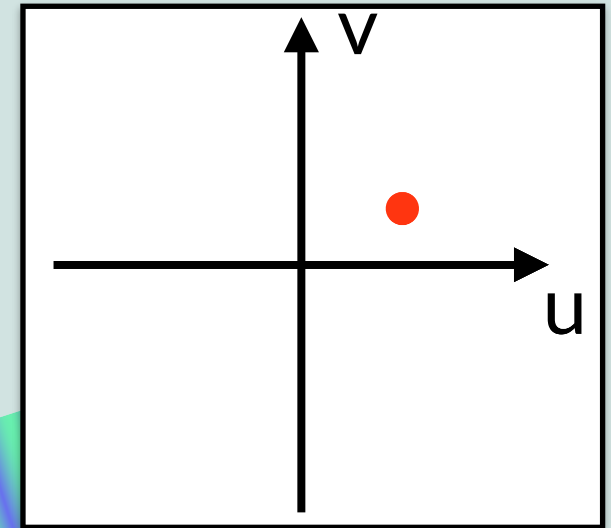
Cygnus A

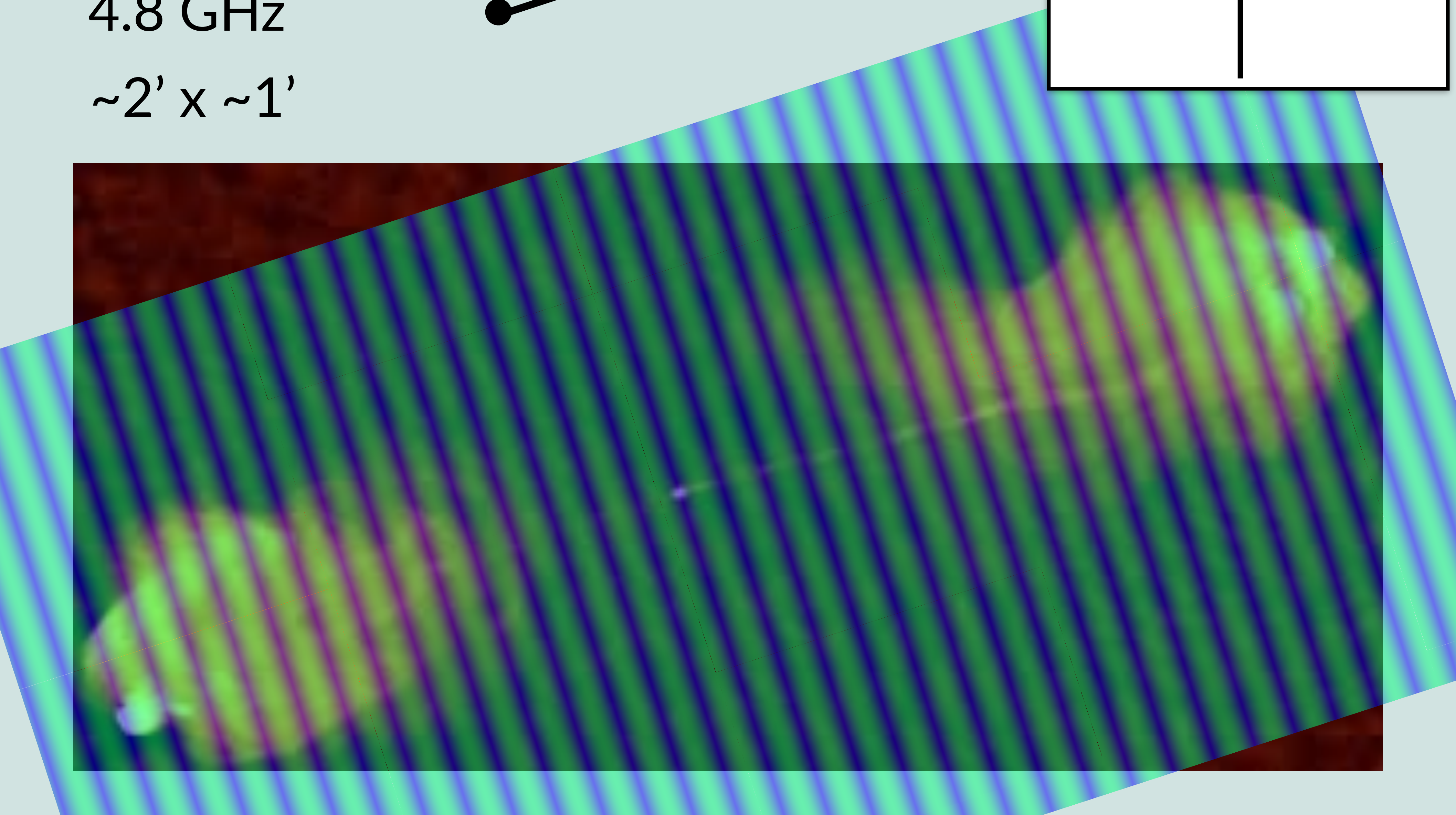
4.8 GHz

~2' x ~1'

$\mathbf{b}_{\mathrm{proj}}$







The 2-element interferometer : uv plane

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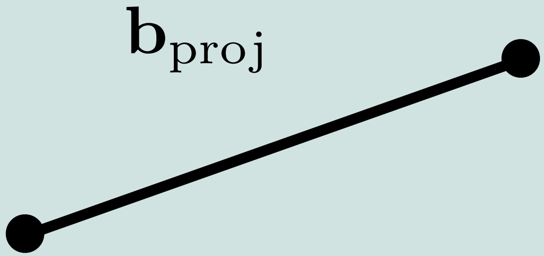

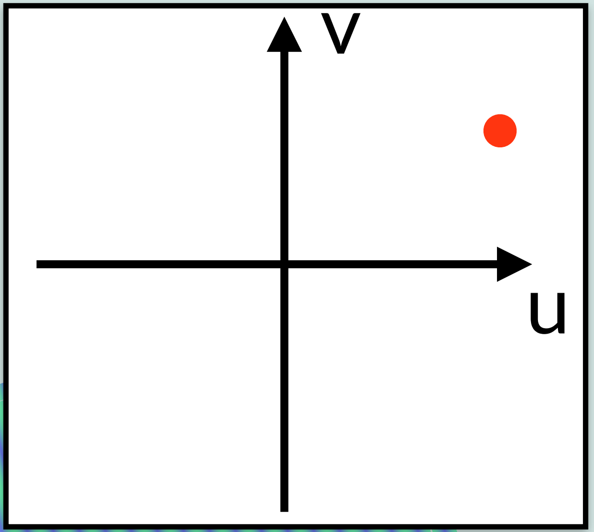

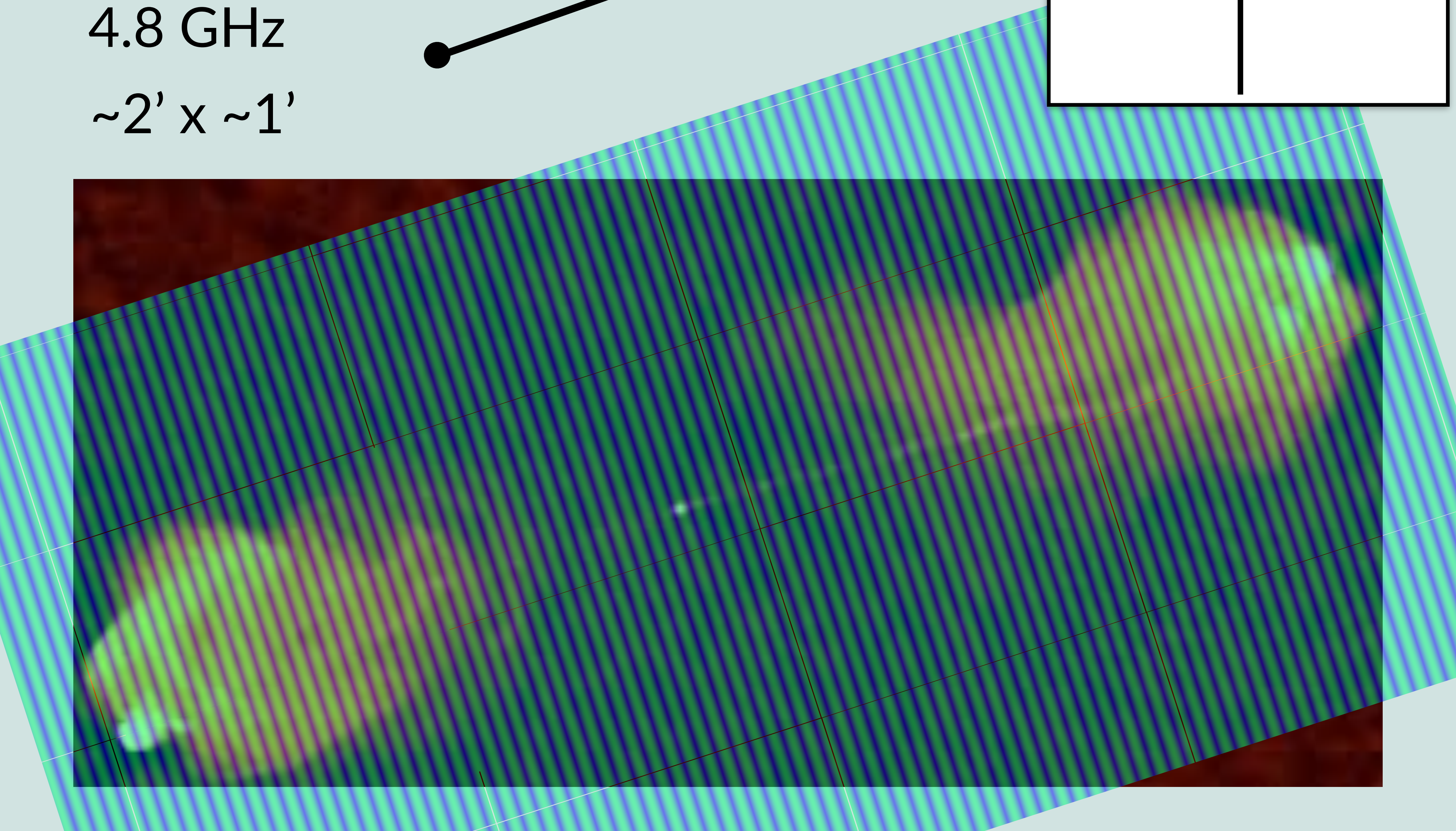

The 2-element interferometer : uv plane

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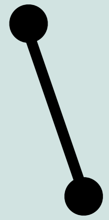

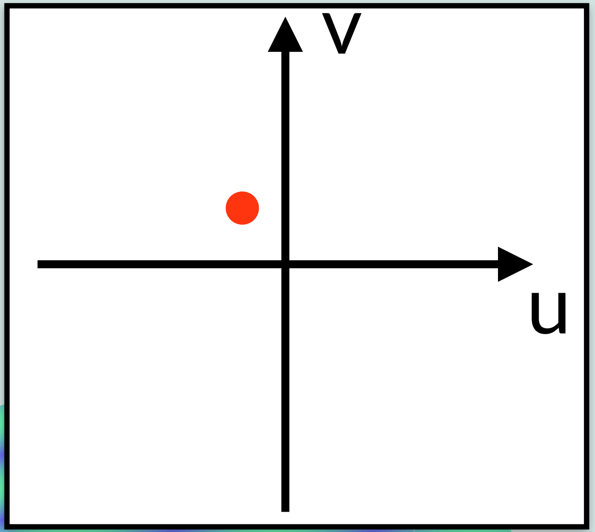

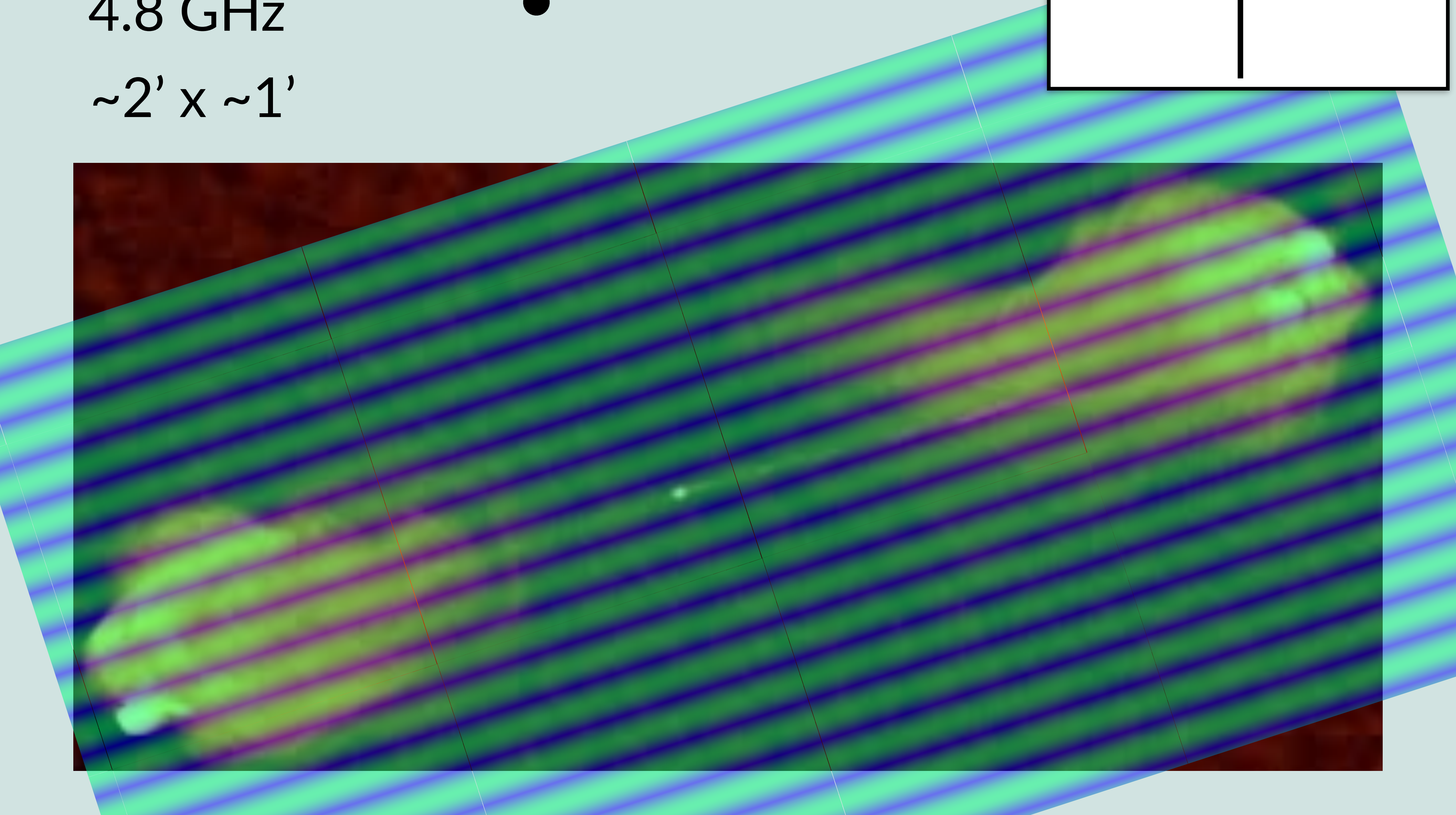

The 2-element interferometer : uv plane

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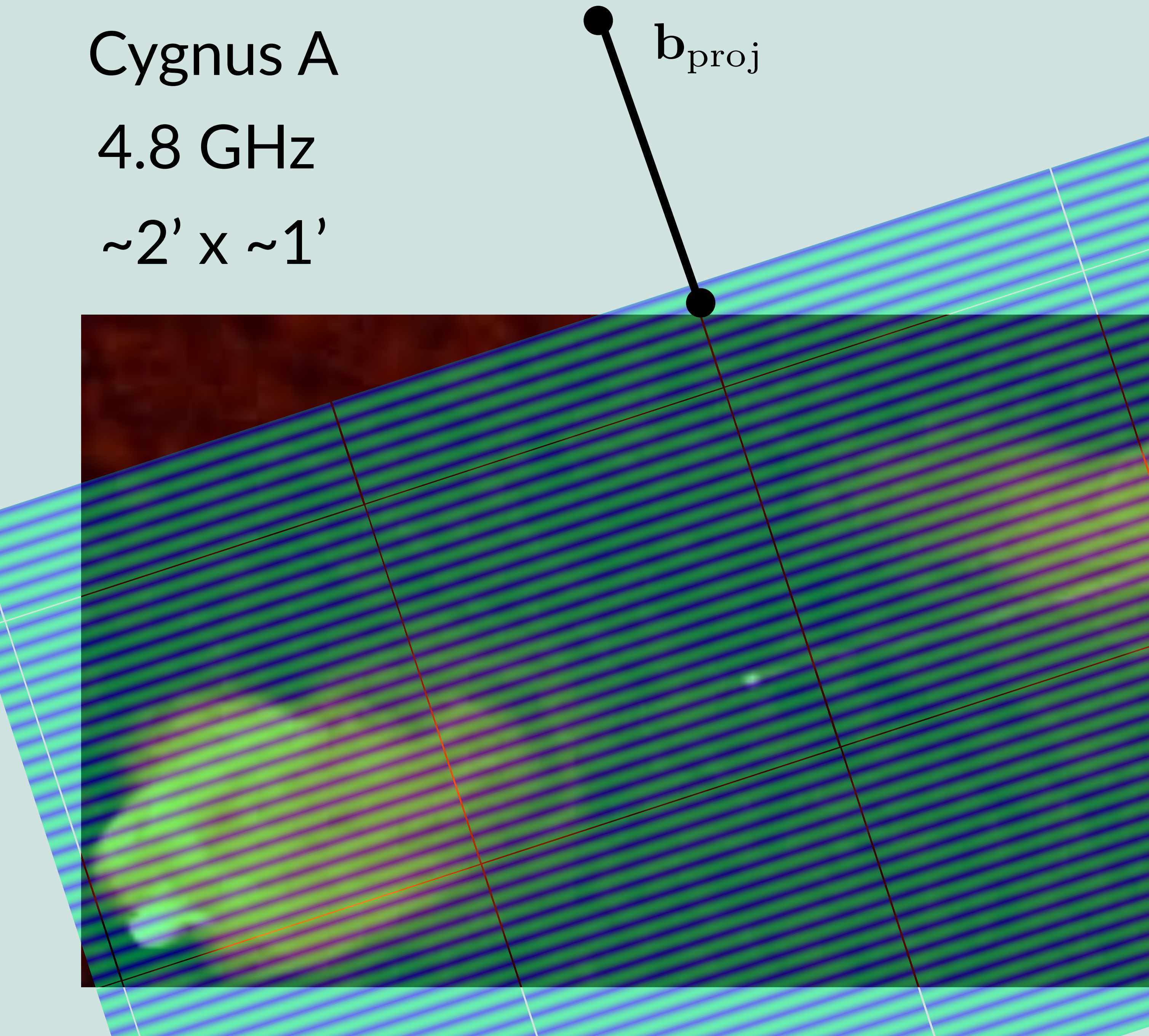
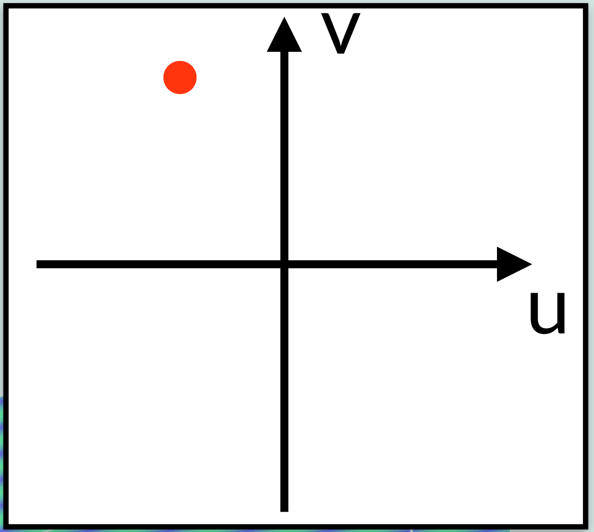
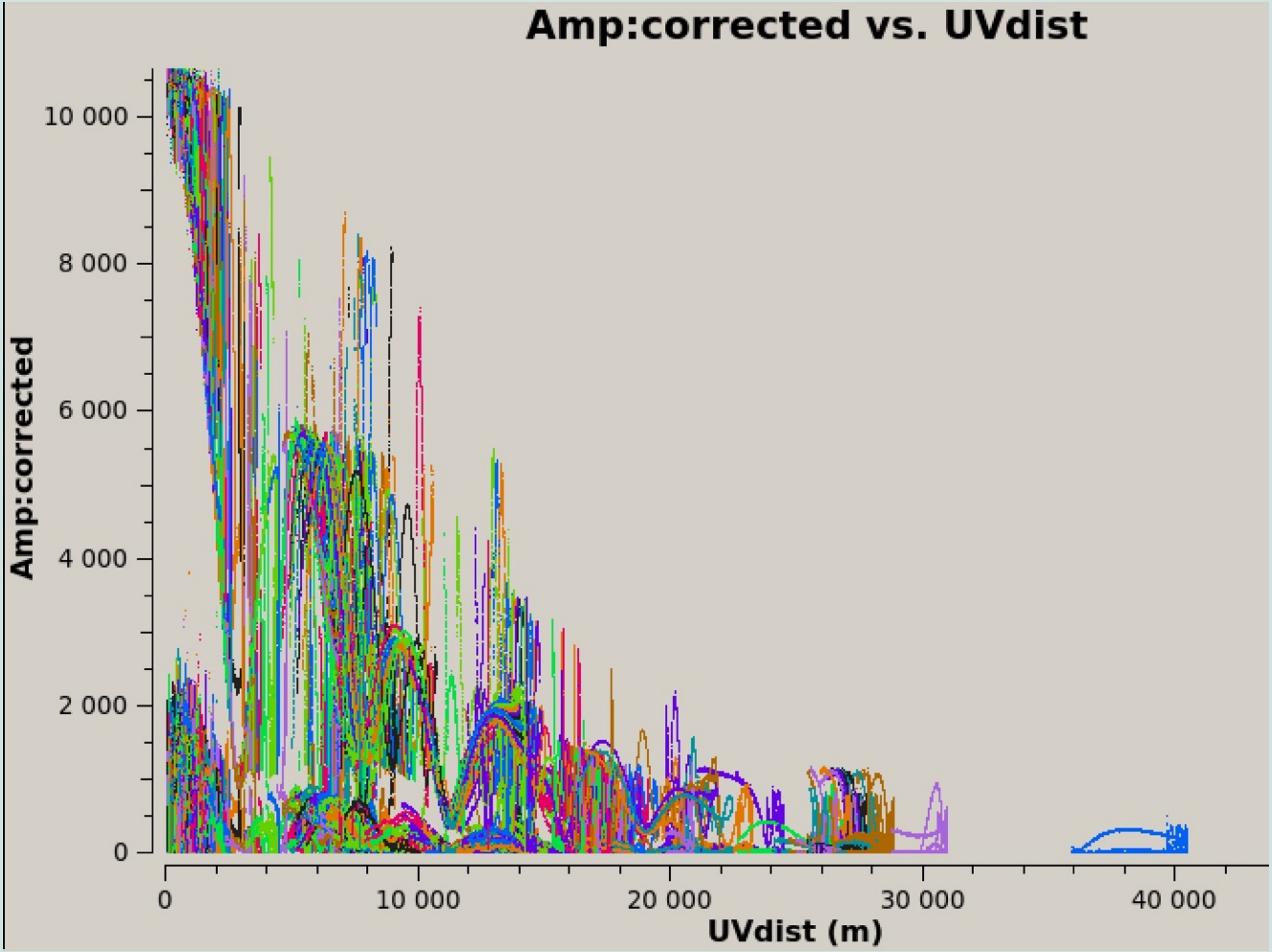




The 2-element interferometer : uv plane



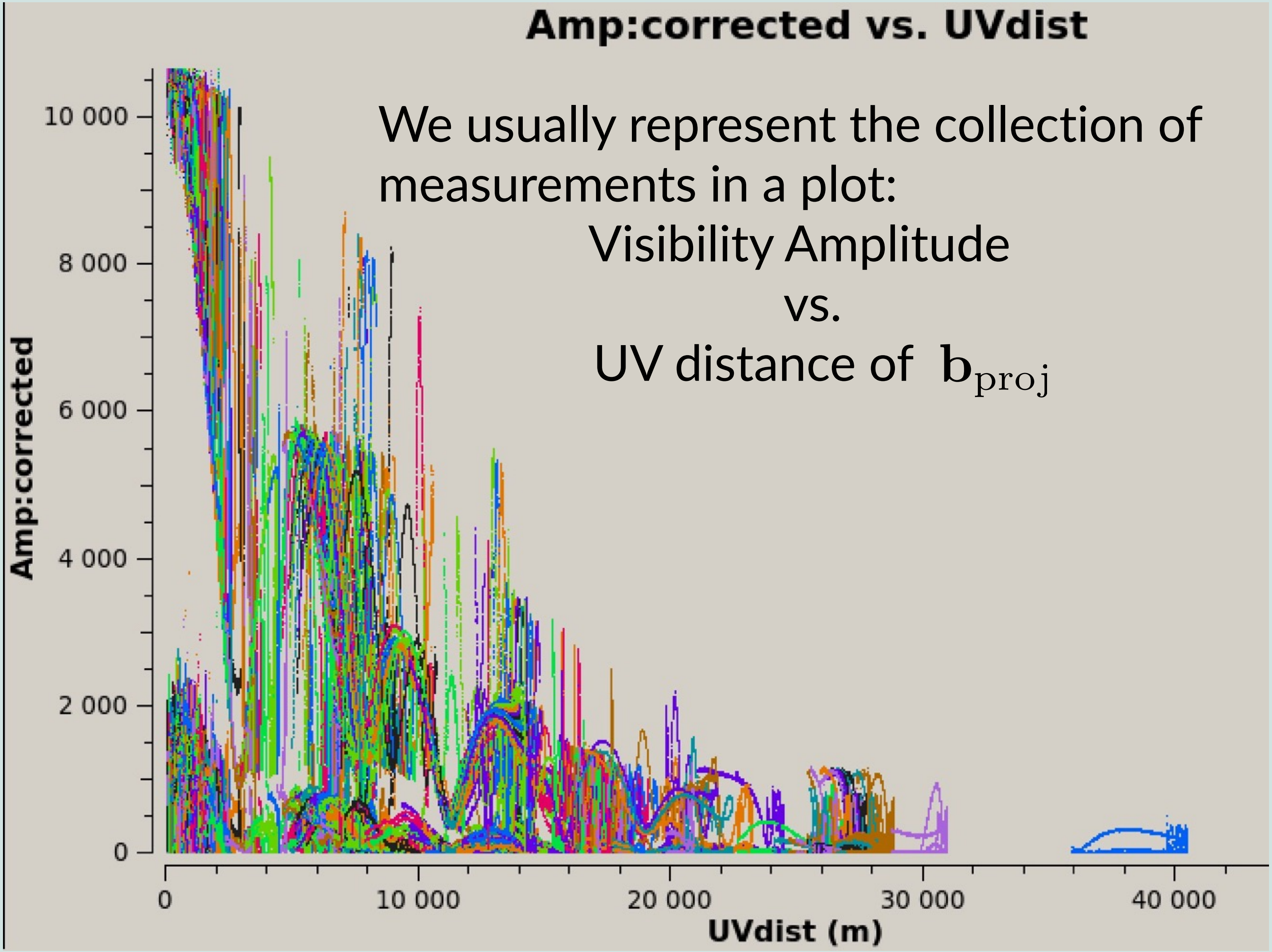
The 2-element interferometer : uv plane

We usually represent the collection of measurements in a plot:

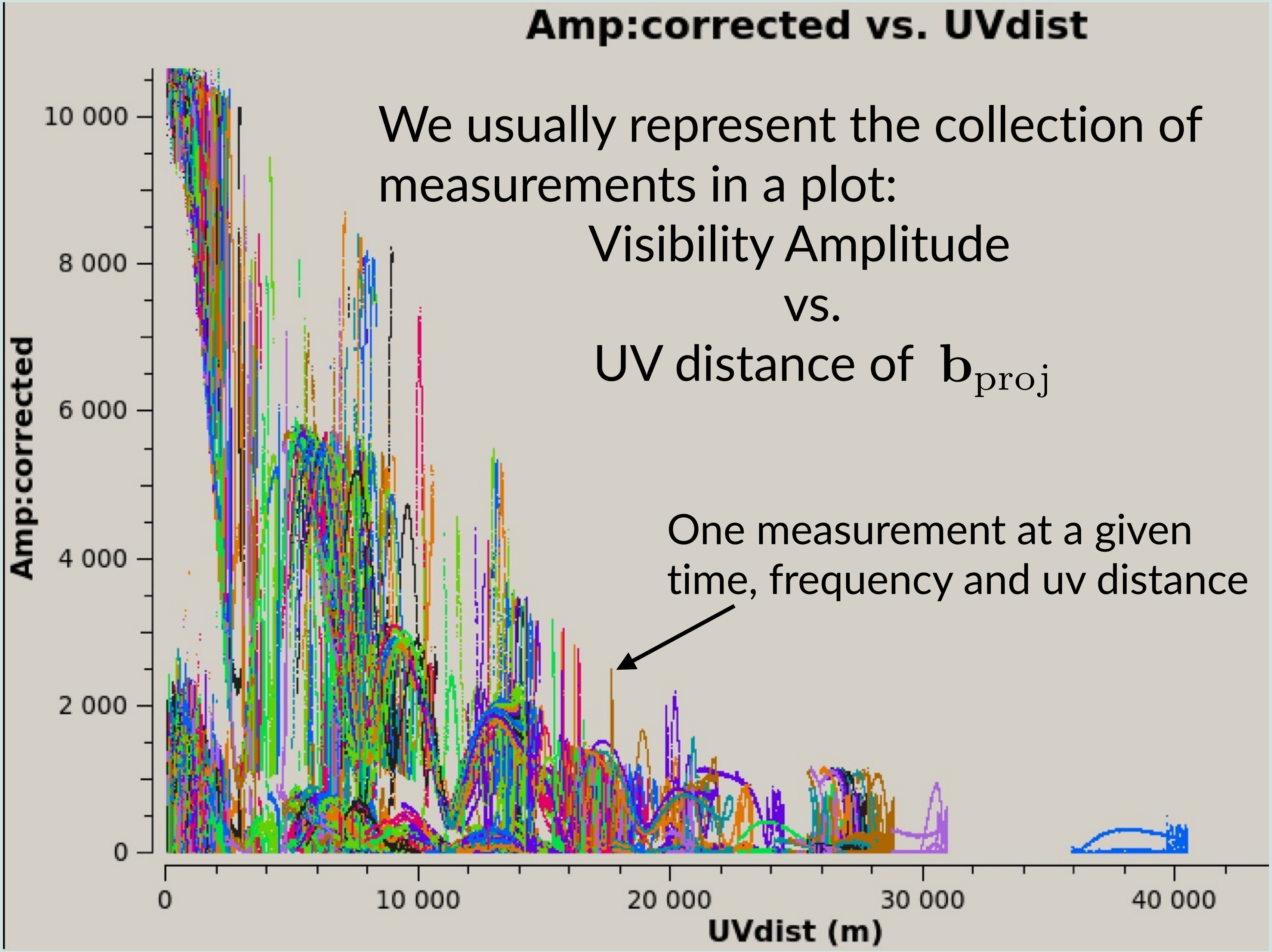
Visibility Amplitude

vs.

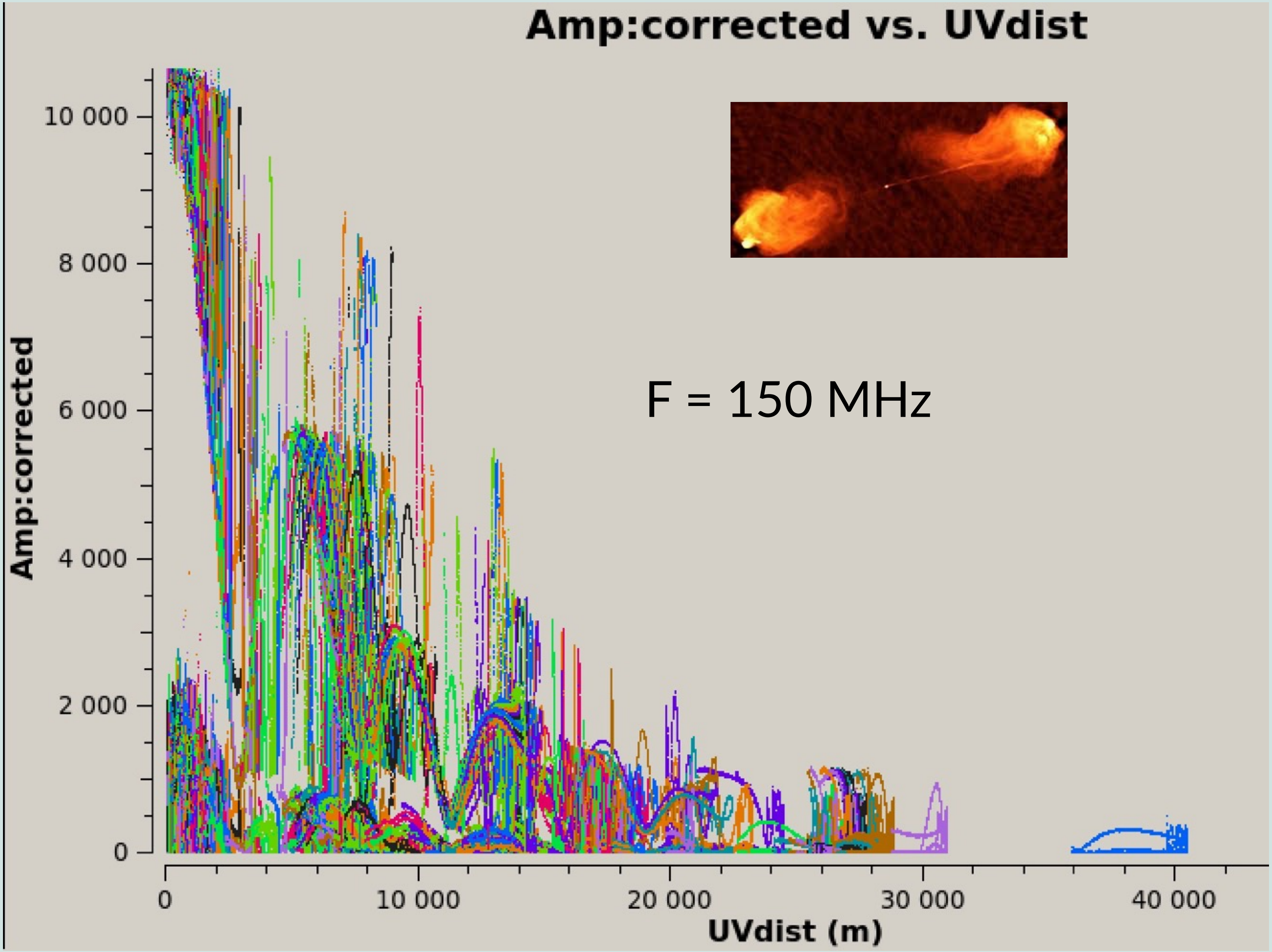
UV distance of $\mathbf{b}_{\mathrm{proj}}$



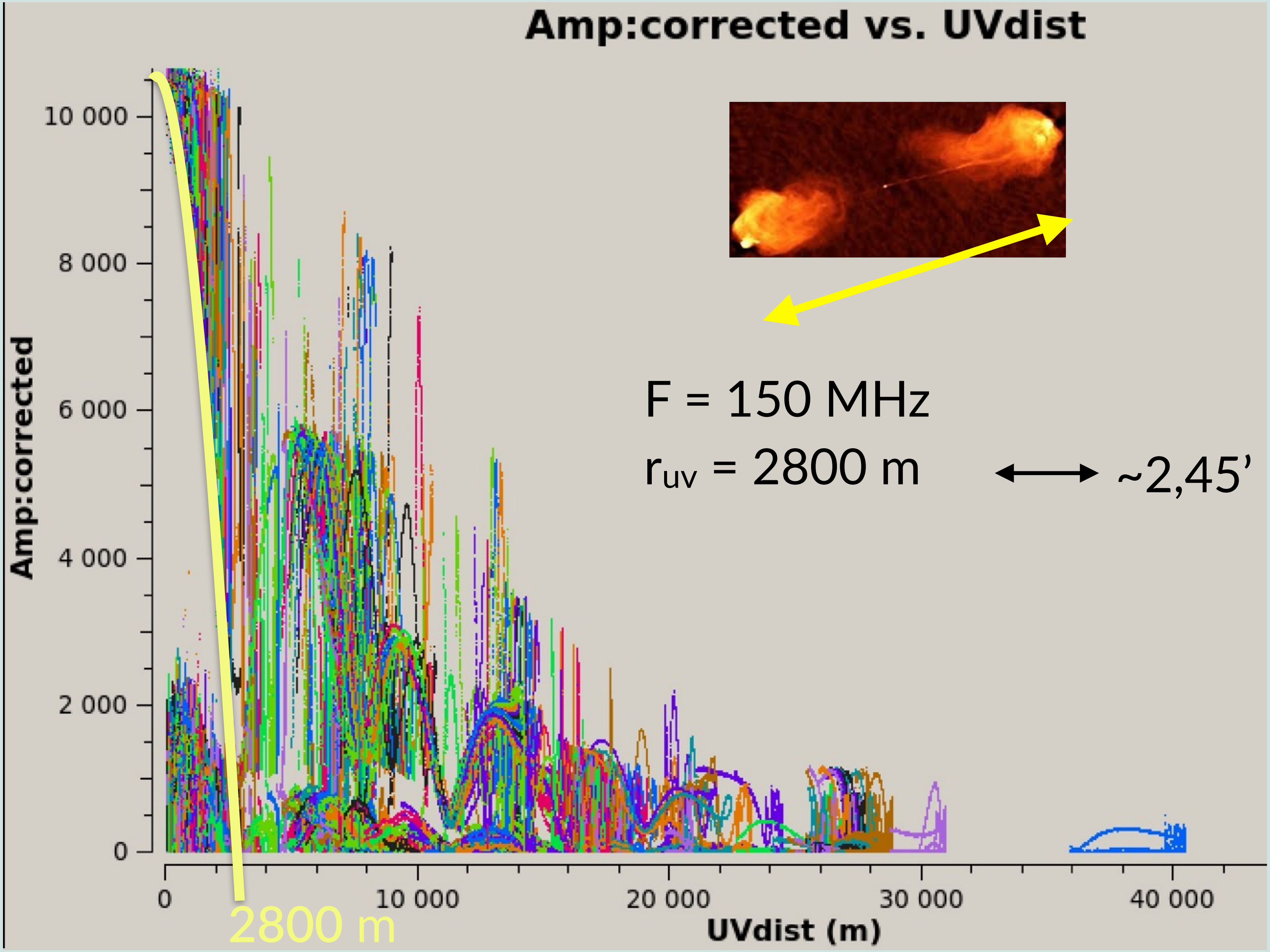
The 2-element interferometer : uv plane



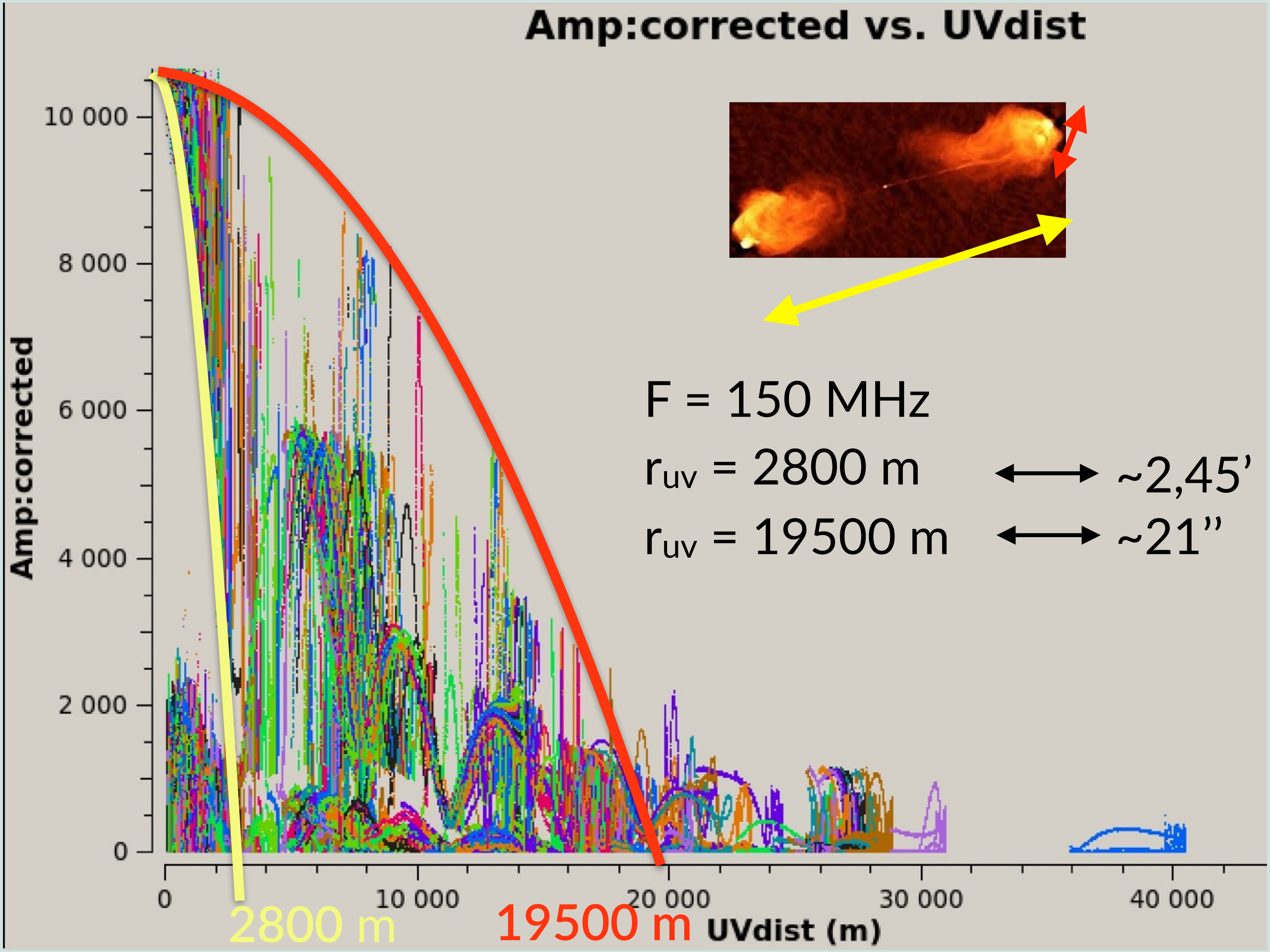
The 2-element interferometer : uv plane



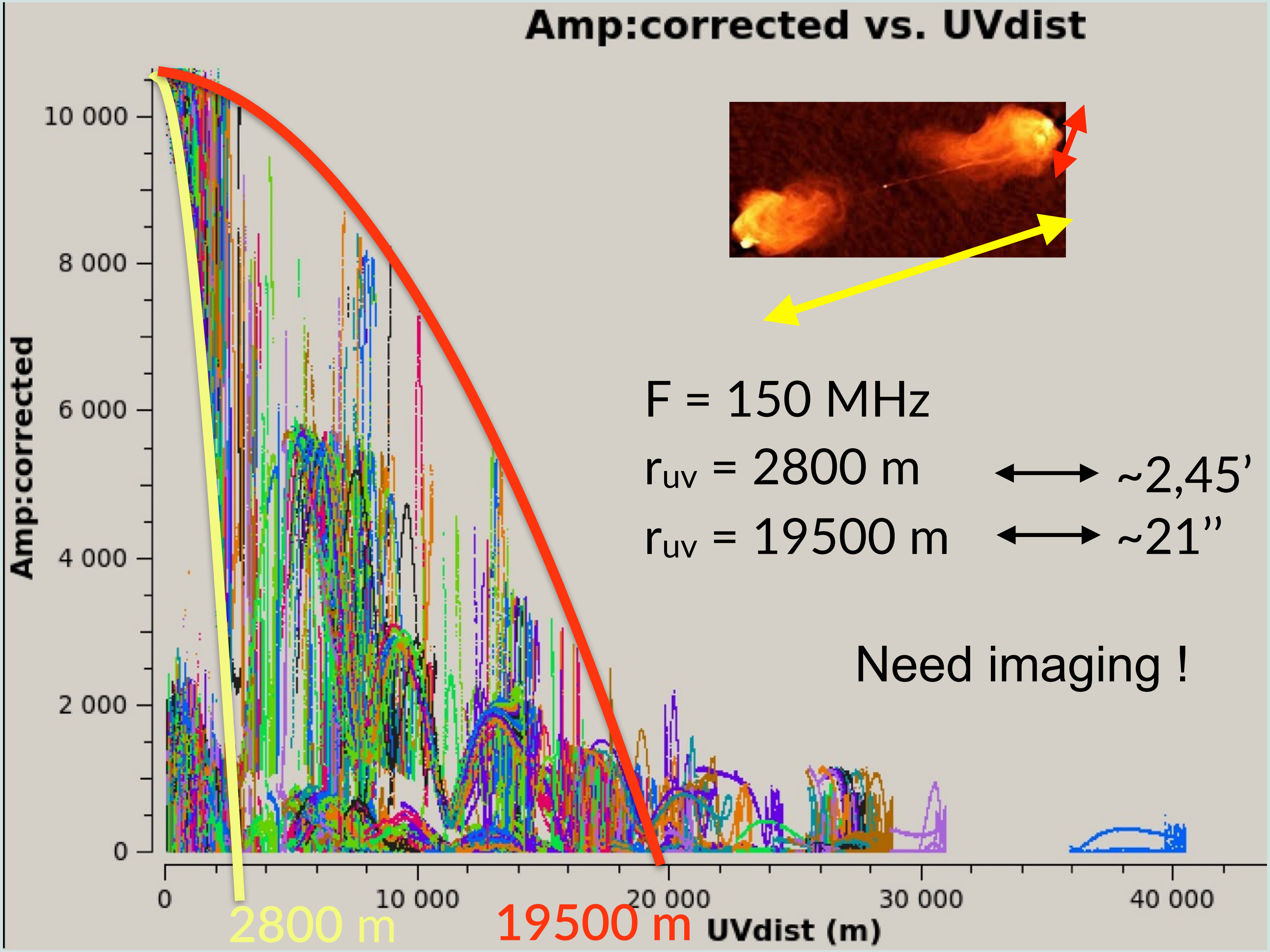
The 2-element interferometer : uv plane



The 2-element interferometer : uv plane



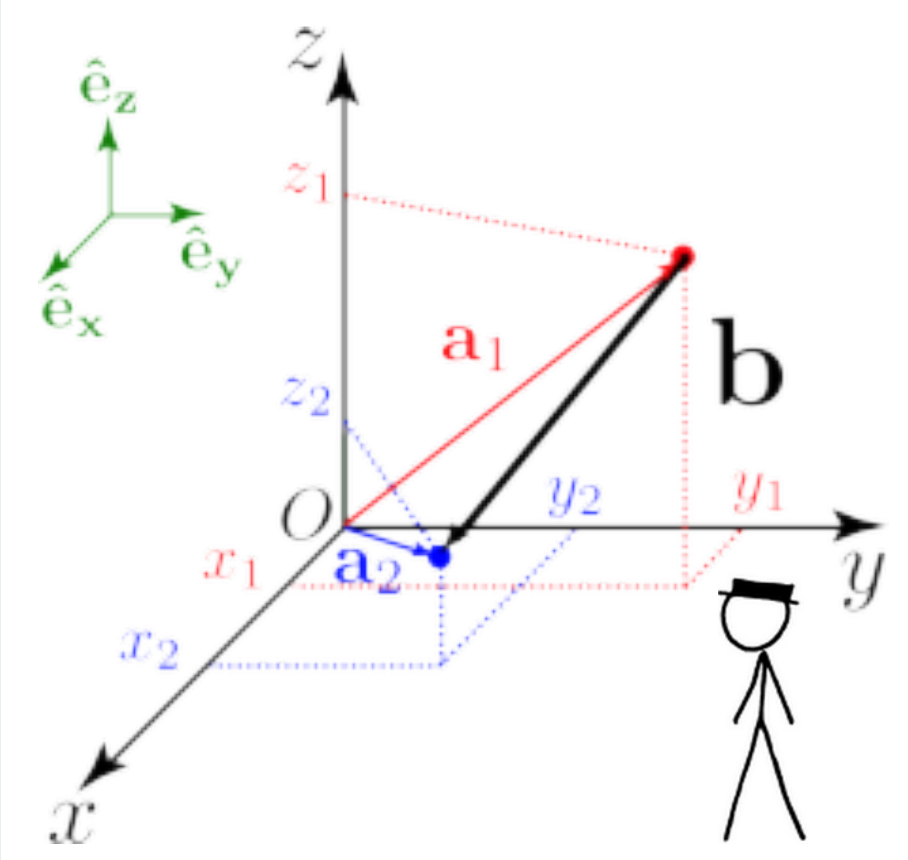
The 2-element interferometer : uv plane



From 1D to 3D baselines

The baseline

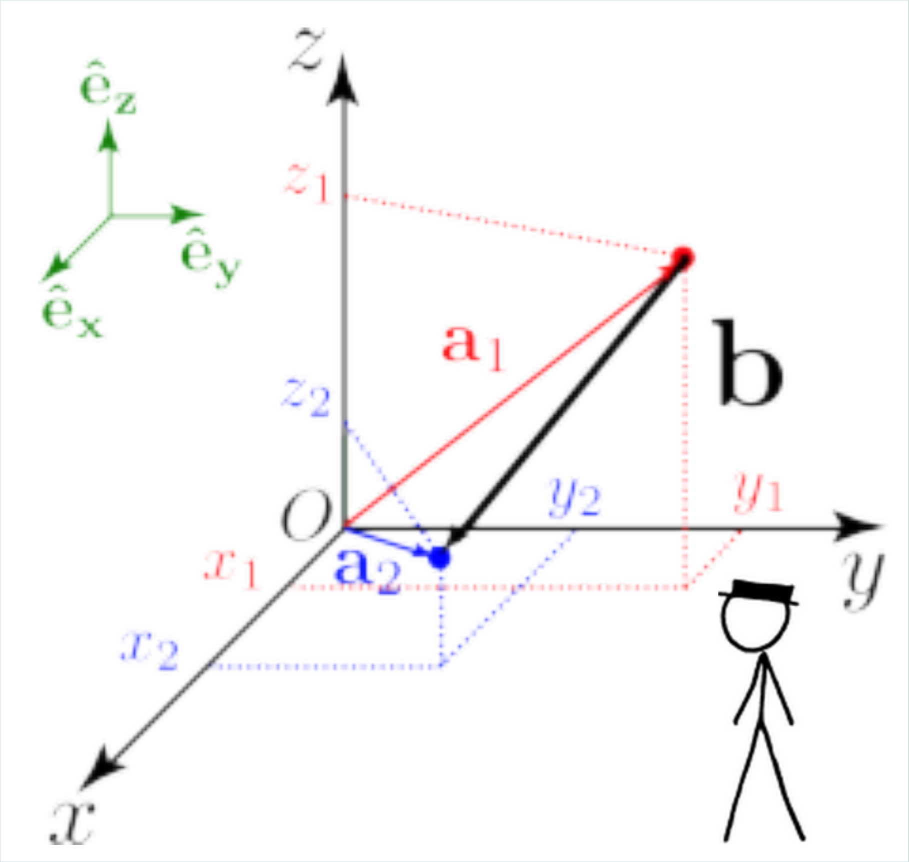
A *baseline* is a vector associated with coordinates in different reference frames

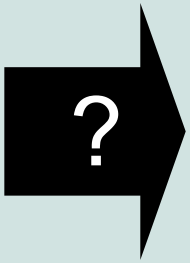


As seen from the ground near the array

The baseline

A *baseline* is a vector associated with coordinates in different reference frames



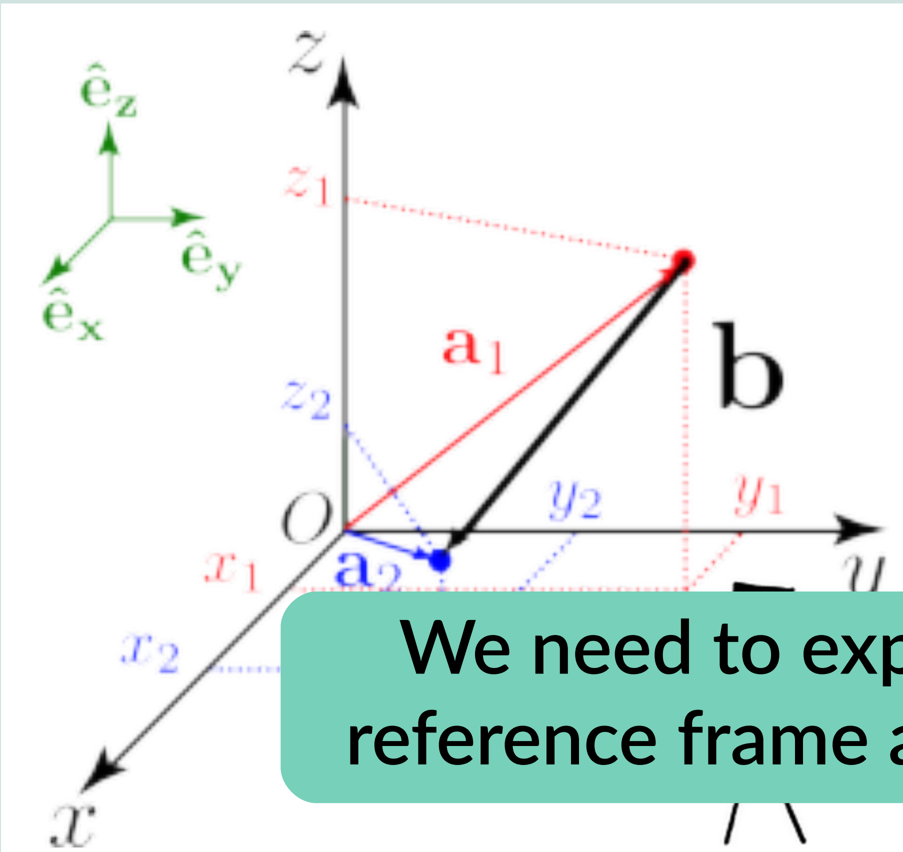


As seen from the ground near the array

As seen from the source, up in the sky

The baseline

A *baseline* is a vector associated with coordinates in different reference frames



?

We need to express the baseline in a reference frame associated with the sky

As seen from the ground near the array

As seen from the source, up in the sky

Baseline and reference frames

We will transform the baseline from the *local* frame

to a *remote* frame

associated with the source we observe

Baseline and reference frames

We will transform the baseline from the *local* frame
to a *remote* frame

associated with the source we observe

Steps:

1) From local (x, y, z) to local (**E**ast, **N**orth, **U**p)

2) From local (**E**,**N**,**U**) to local azimuth/Elevation ($\mathcal{A}, \mathcal{E}$)

3) From local ($\mathcal{A}, \mathcal{E}$) to equatorial (H, δ) or (**X**,**Y**,**Z**)

4) From equatorial (**X**,**Y**,**Z**) to the (u, v, w)-space

Baseline and reference frames

We will transform the baseline from the *local* frame
to a *remote* frame
associated with the source we observe

Steps:

1) From local (x, y, z) to local (**E**ast, **N**orth, **U**p)

2) From local (**E**,**N**,**U**) to local azimuth/Elevation ($\mathcal{A}, \mathcal{E}$)

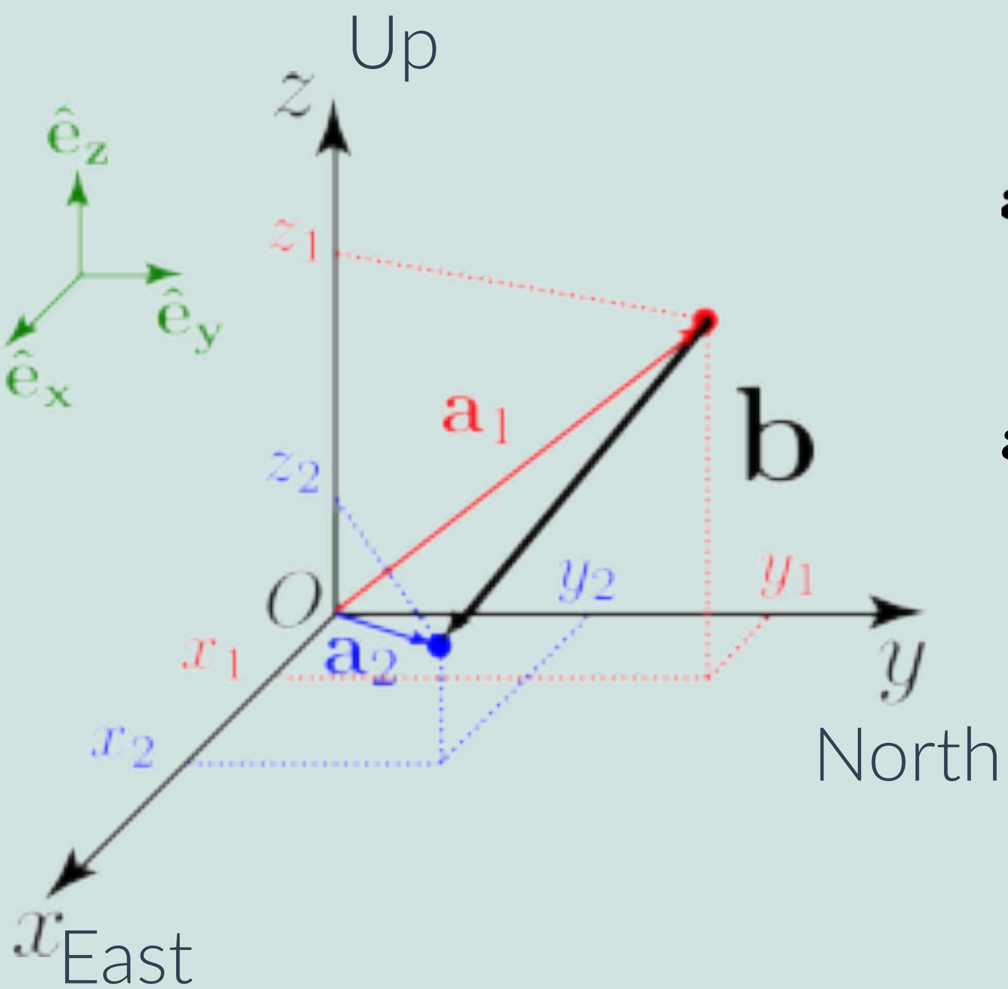
3) From local ($\mathcal{A}, \mathcal{E}$) to equatorial (H, δ) or (**X**,**Y**,**Z**)

4) From equatorial (**X**,**Y**,**Z**) to the (u, v, w)-space

Then,
we will be ready to express the baseline

Step 1: From local (x, y, z) to local (East, North, Up)

The first transform is trivial as we can map the x, y, z axes to the East, North, Up axes, introducing the cardinal directions.



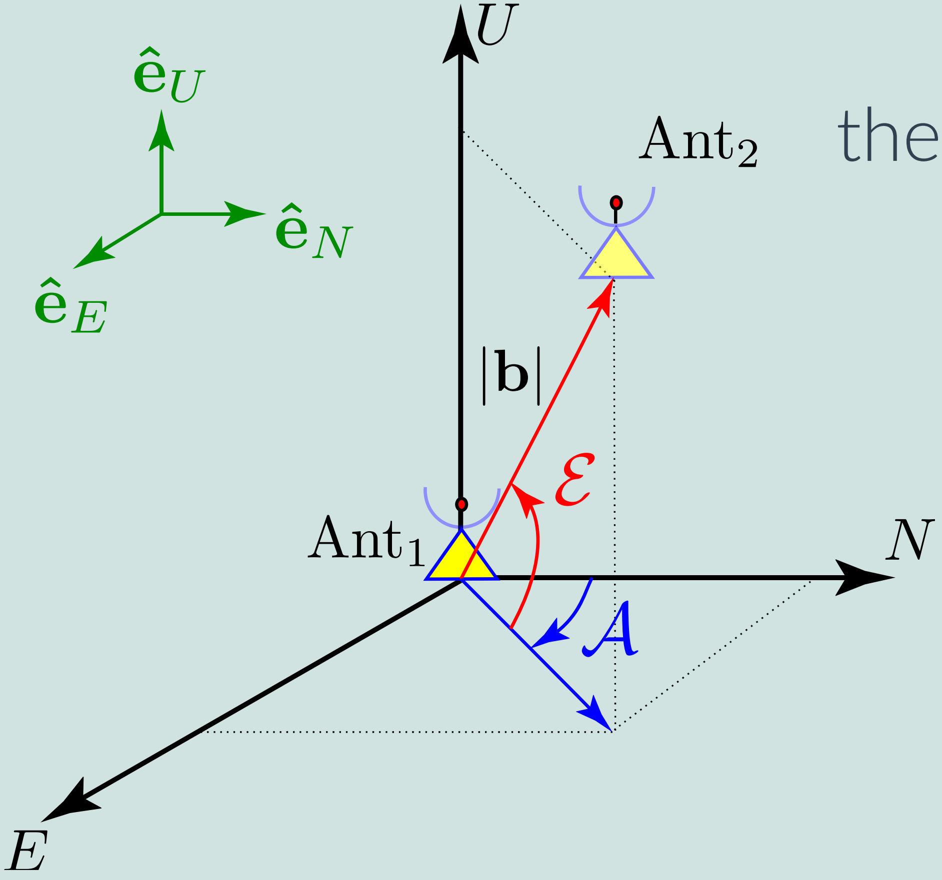
$$\mathbf{a}_1 = \vec{OA}_1 = x_1\hat{\mathbf{e}}_{\mathbf{x}} + y_1\hat{\mathbf{e}}_{\mathbf{y}} + z_1\hat{\mathbf{e}}_{\mathbf{z}}$$
$$= x_1'\hat{\mathbf{e}}_{\mathbf{E}} + y_1'\hat{\mathbf{e}}_{\mathbf{N}} + z_1'\hat{\mathbf{e}}_{\mathbf{U}}$$
$$\mathbf{a}_2 = \vec{OA}_2 = x_2\hat{\mathbf{e}}_{\mathbf{x}} + y_2\hat{\mathbf{e}}_{\mathbf{y}} + z_2\hat{\mathbf{e}}_{\mathbf{z}}$$
$$= x_2'\hat{\mathbf{e}}_{\mathbf{E}} + y_2'\hat{\mathbf{e}}_{\mathbf{N}} + z_2'\hat{\mathbf{e}}_{\mathbf{U}}$$

$$\mathbf{b} = \mathbf{a}_2 - \mathbf{a}_1$$
$$\mathbf{b} = b_1\hat{\mathbf{e}}_{\mathbf{N}} + b_2\hat{\mathbf{e}}_{\mathbf{N}} + b_3\hat{\mathbf{e}}_{\mathbf{U}}$$

Step 2: From local (East, North, Up) to Azimuth/Elevation

The second transform is to express the cartesian coordinates as a function of:

the Azimuth	$\mathcal{A}$	Origin:	North	(astro)
			South	(geo)
the Elevation	$\mathcal{E}$	Origin:	Local horizon	



Step 2: From local (East, North, Up) to Azimuth/Elevation

The second transform is to express the cartesian coordinates as a function of:

the Azimuth	$\mathcal{A}$	Origin:	North South	(astro) (geo)
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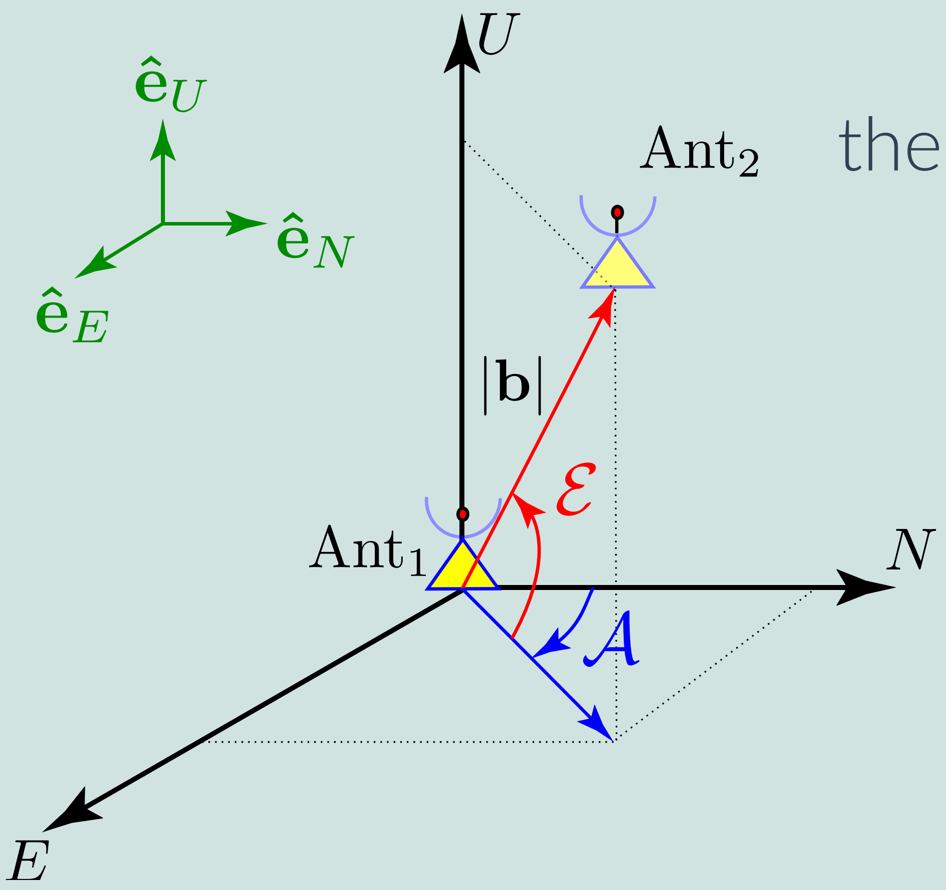
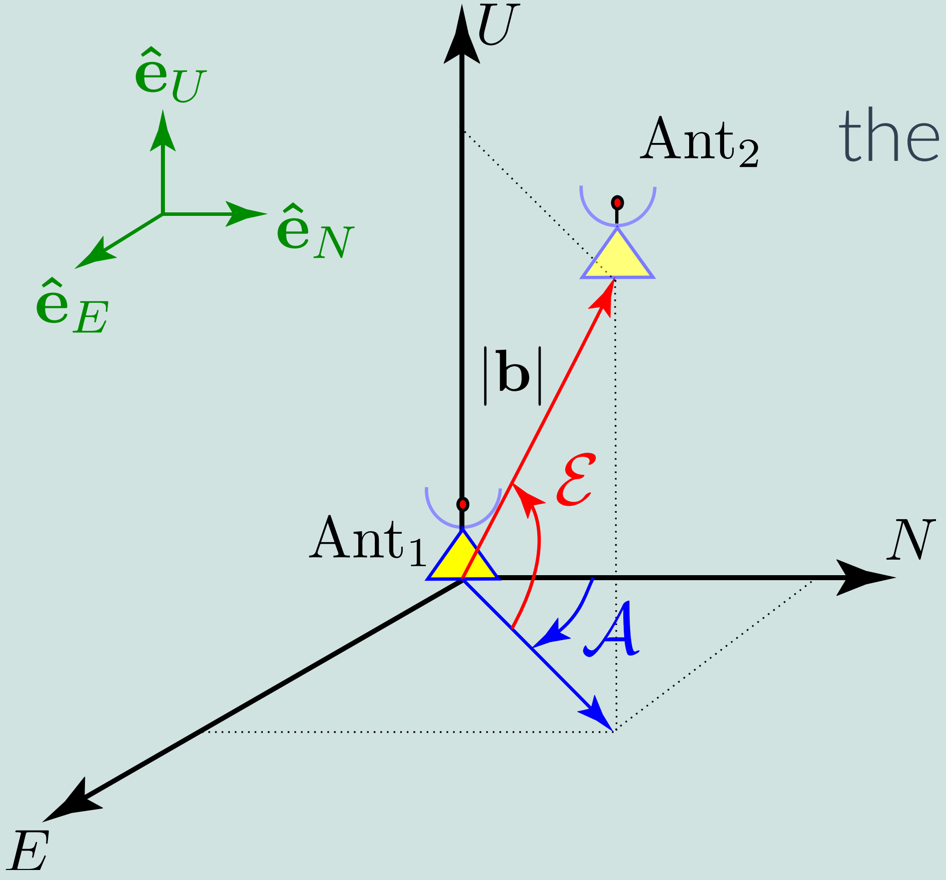


$$\mathbf{b}_{\mathrm{ENU}} = |\mathbf{b}| \begin{bmatrix} \sin\mathcal{A}\cos\mathcal{E} \\ \cos\mathcal{A}\cos\mathcal{E} \\ \sin\mathcal{E} \end{bmatrix}$$

Step 2: From local (East, North, Up) to Azimuth/Elevation

The second transform is to express the cartesian coordinates as a function of:

the Azimuth	$\mathcal{A}$	Origin:	North	(astro)
			South	(geo)
the Elevation	$\mathcal{E}$	Origin:	Local horizon	



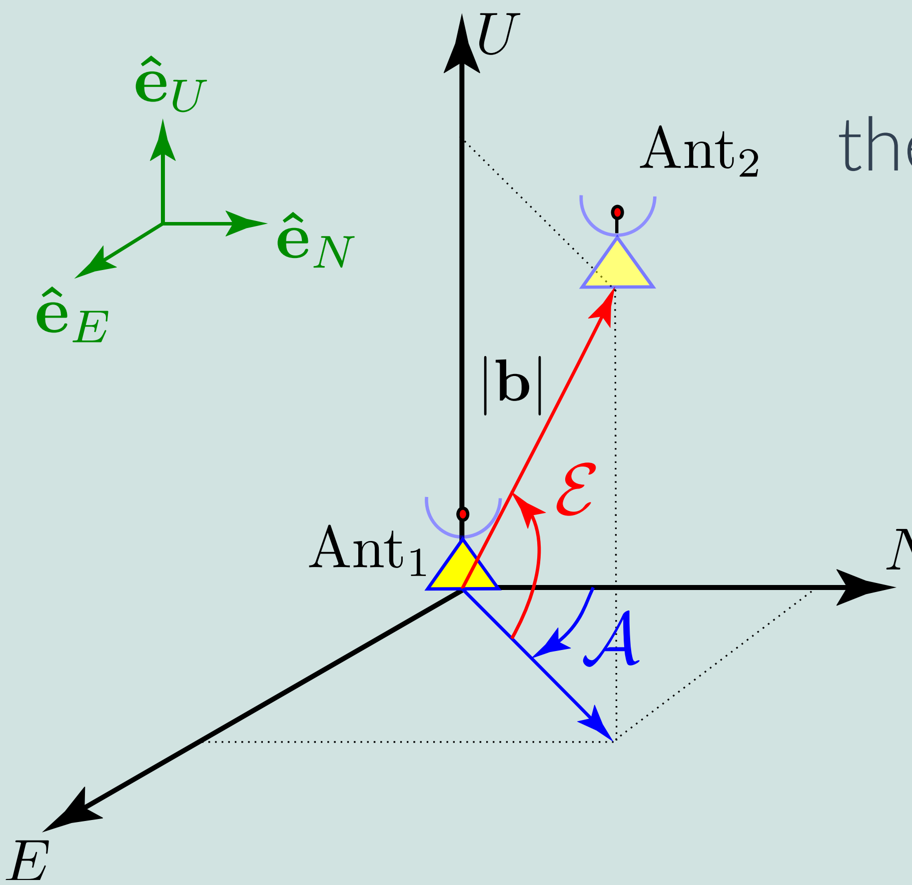
Length of the baseline $\sqrt{b_1^2 + b_2^2 + b_3^2}$

$$\mathbf{b}_{\mathrm{ENU}} = |\mathbf{b}| \begin{bmatrix} \sin\mathcal{A}\cos\mathcal{E} \\ \cos\mathcal{A}\cos\mathcal{E} \\ \sin\mathcal{E} \end{bmatrix}$$

Step 2: From local (East, North, Up) to Azimuth/Elevation

The second transform is to express the cartesian coordinates as a function of:

the Azimuth	$\mathcal{A}$	Origin:	North	(astro)
			South	(geo)
the Elevation	$\mathcal{E}$	Origin:	Local horizon	



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$$\mathbf{b}_{\mathrm{ENU}} = |\mathbf{b}| \begin{bmatrix} \sin\mathcal{A}\cos\mathcal{E} \\ \cos\mathcal{A}\cos\mathcal{E} \\ \sin\mathcal{E} \end{bmatrix}$$

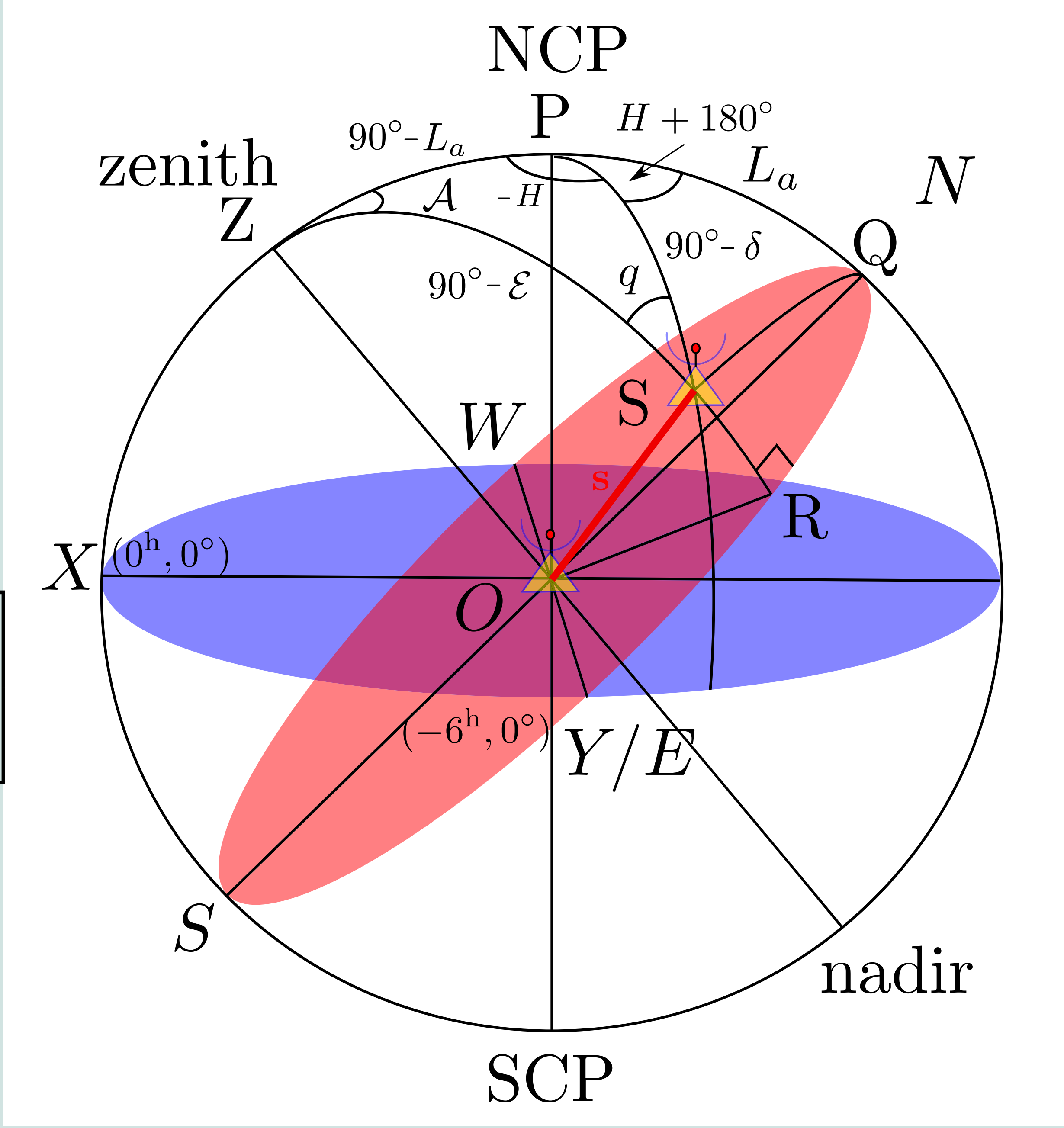
Trigonometry leads to the inverse expressions of $\mathcal{A}$ and $\mathcal{E}$ as a function of the (E,N,U) coordinates

Step 3: From $\mathcal{A}$, $\mathcal{E}$ to equatorial coordinates

The second transform is to express the cartesian coordinates as a function of: (δ, H)

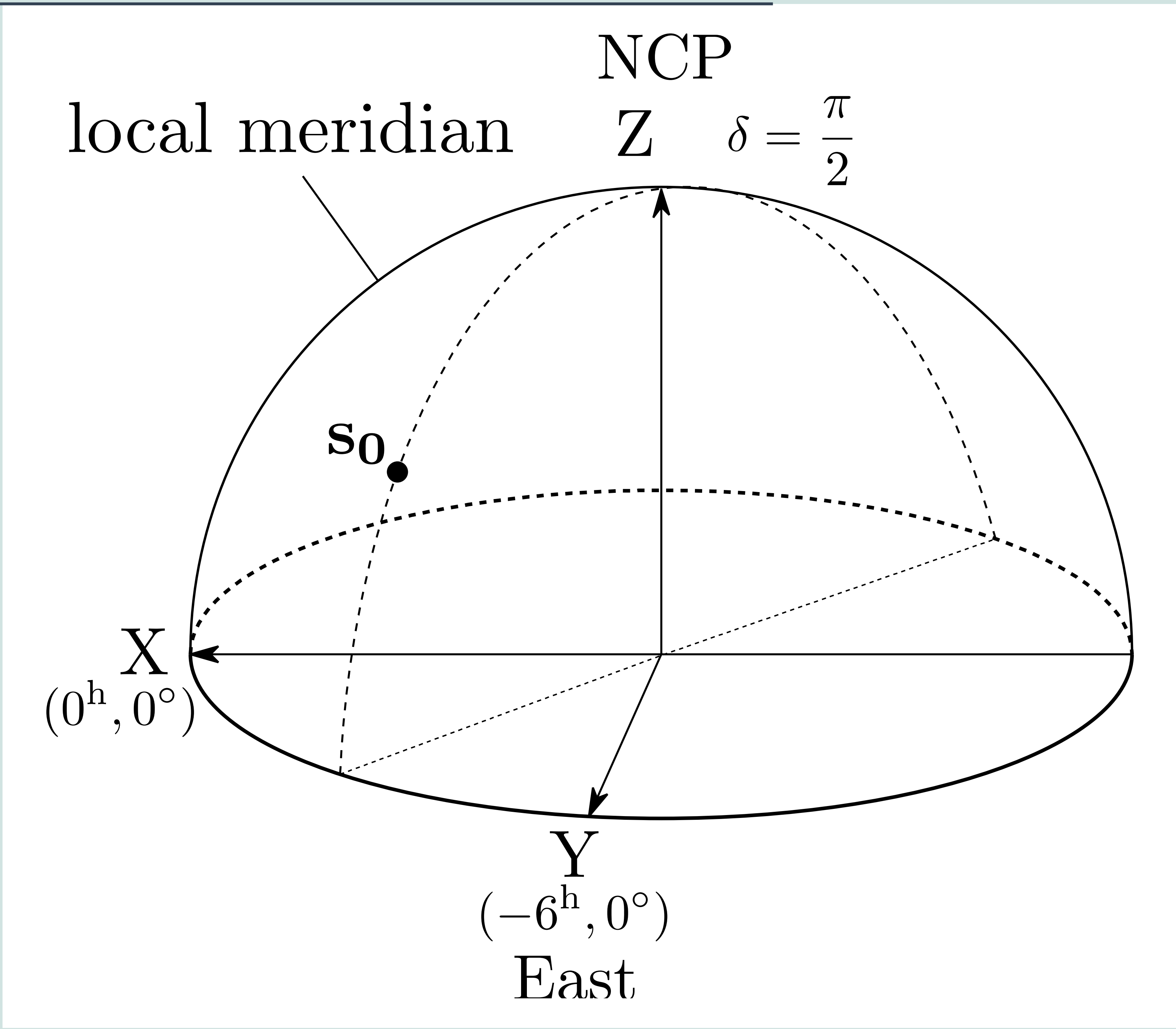
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} |\mathbf{b}| \cos\delta \cos H \\ -|\mathbf{b}| \cos\delta \sin H \\ |\mathbf{b}| \sin\delta \end{bmatrix}$$

$$= |\mathbf{b}| * \begin{bmatrix} \cos L_a \sin\mathcal{E} - \sin L_a \cos\mathcal{E} \cos\mathcal{A} \\ \cos E \sin\mathcal{A} \\ \sin L_a \sin\mathcal{E} + \cos L_a \cos\mathcal{E} \cos\mathcal{A} \end{bmatrix}$$

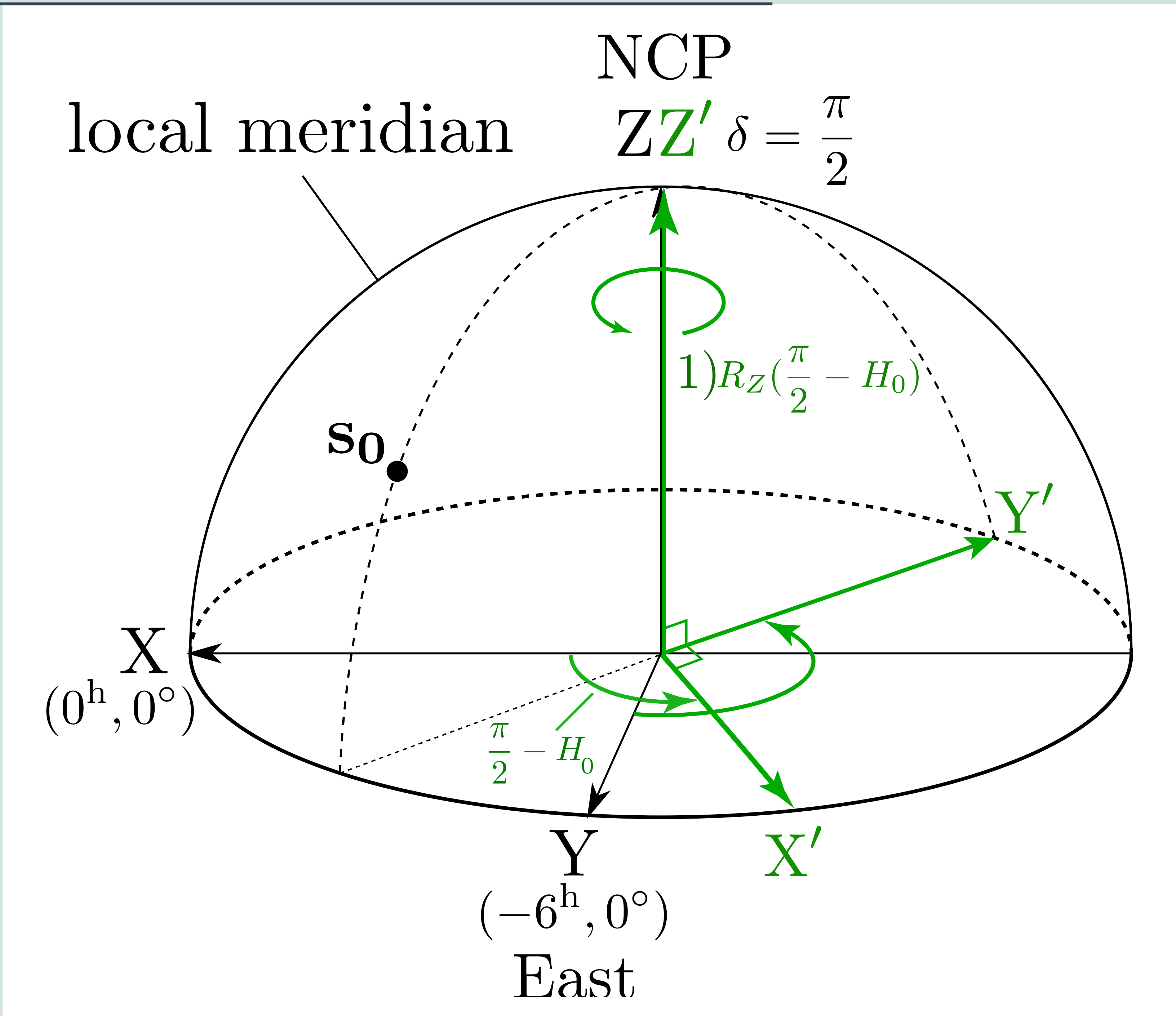


Full derivation in Appendix

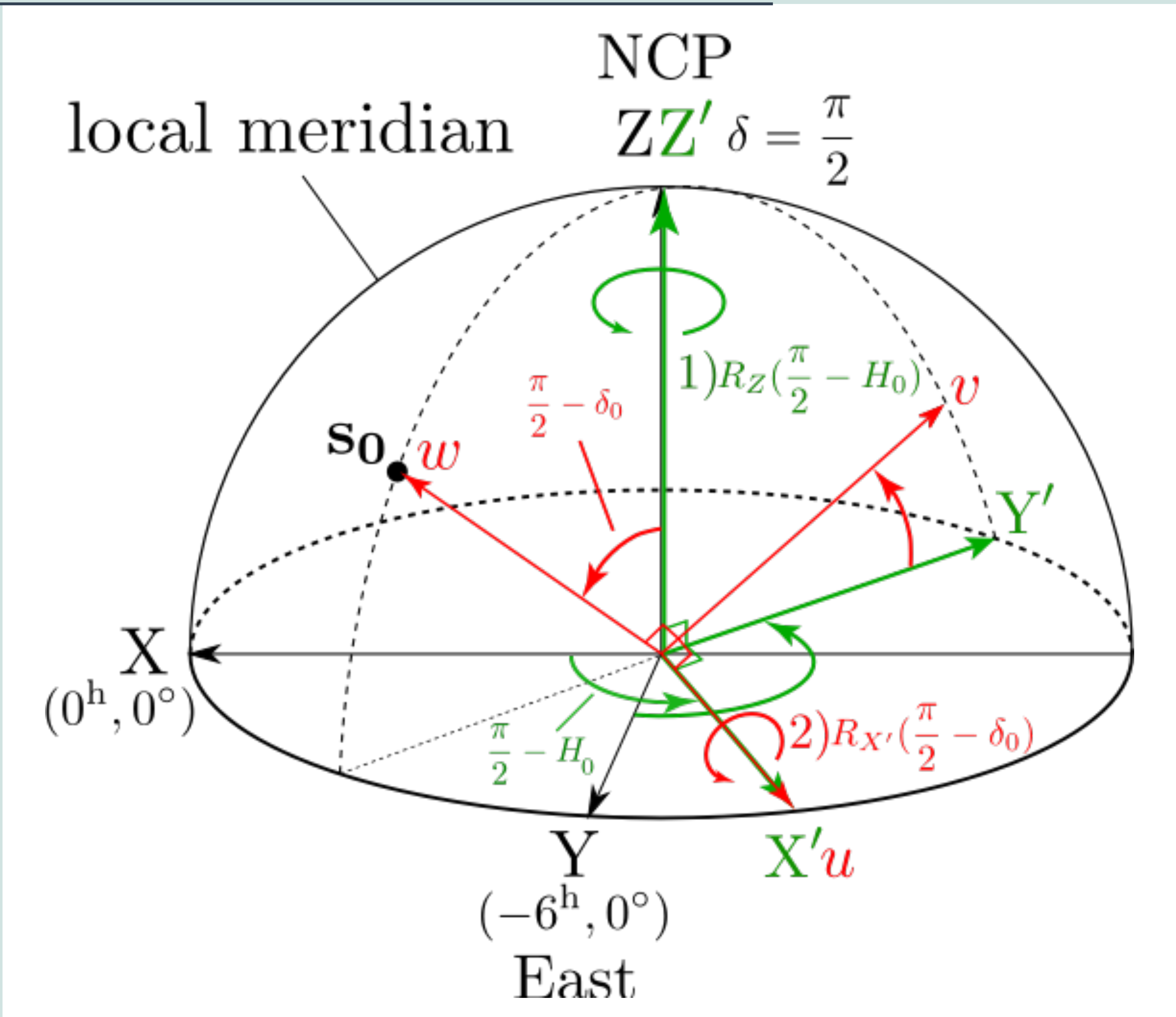
Step 4: From $\mathcal{A}$, $\mathcal{E}$ to equatorial coordinates



Step 4: From $\mathcal{A}$, $\mathcal{E}$ to equatorial coordinates



Step 4: From $\mathcal{A}$, $\mathcal{E}$ to equatorial coordinates



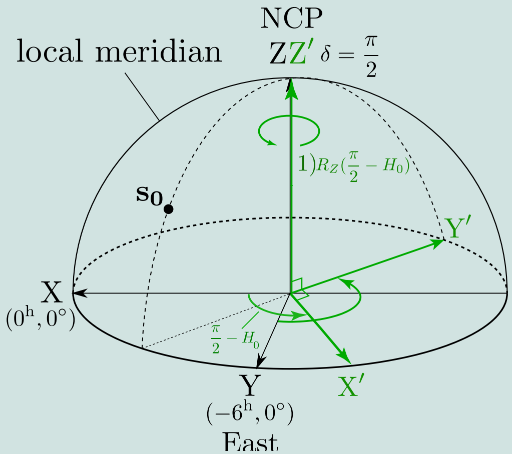
Step 4: From $\mathcal{A}$, $\mathcal{E}$ to equatorial coordinates

$$\mathbf{b}_\lambda = \frac{\mathbf{b}}{\lambda} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \qquad \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Step 4: From $\mathcal{A}$, $\mathcal{E}$ to equatorial coordinates

$$\mathbf{b}_\lambda = \frac{\mathbf{b}}{\lambda} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \qquad \mathcal{R}_{\mathrm{Z}}^{\frac{\pi}{2}-H_0} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

1



Step 4: From $\mathcal{A}$, $\mathcal{E}$ to equatorial coordinates

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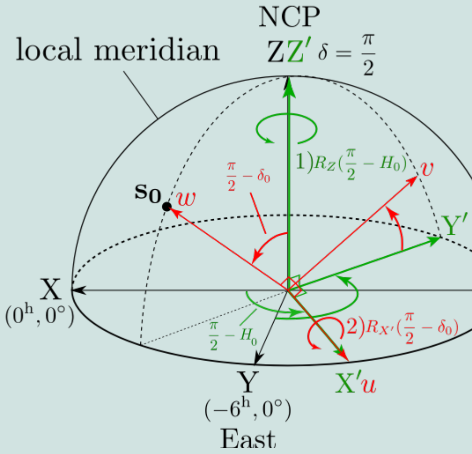
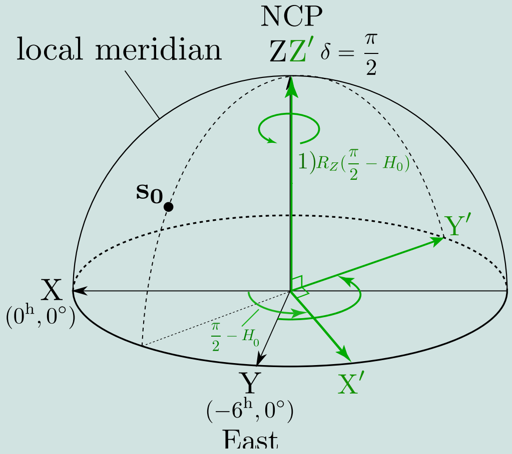
2

1



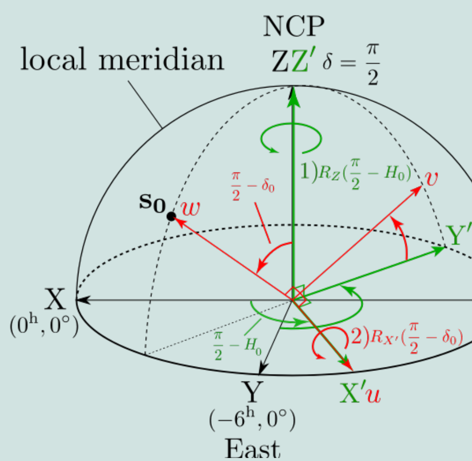


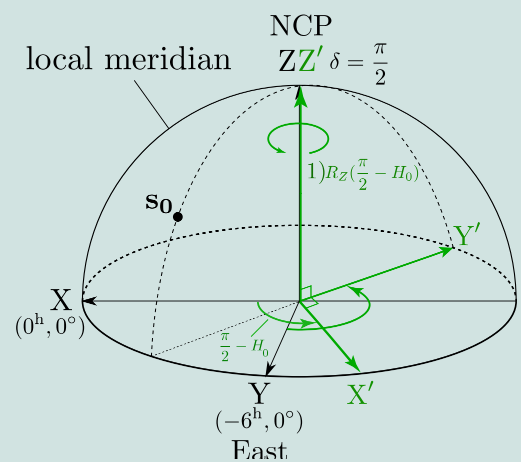
Step 4: From $\mathcal{A}$, $\mathcal{E}$ to equatorial coordinates

Order of operations

$$\mathbf{b}_\lambda = \frac{\mathbf{b}}{\lambda} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \mathcal{R}_Z^{\frac{\pi}{2}-H_0}\ \mathcal{R}_Z^{\frac{\pi}{2}-H_0} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

2 1



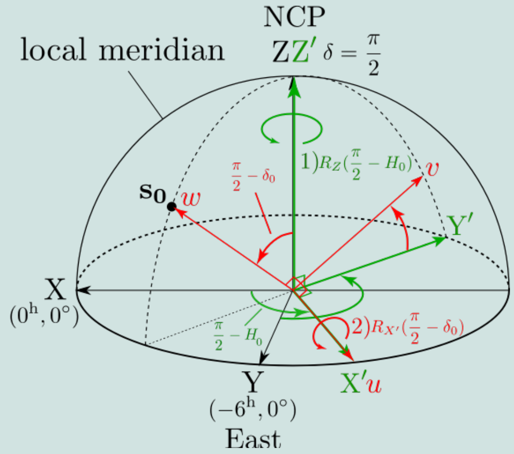


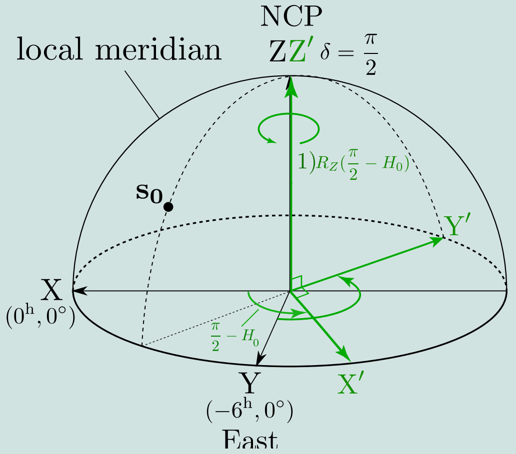
Step 4: From $\mathcal{A}$, $\mathcal{E}$ to equatorial coordinates

Order of operations

$$\mathbf{b}_\lambda = \frac{\mathbf{b}}{\lambda} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \mathcal{R}_Z^{\frac{\pi}{2}-H_0} \; \mathcal{R}_Z^{\frac{\pi}{2}-H_0} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

2 1





$$\mathbf{b}_\lambda = \frac{\mathbf{b}}{\lambda} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} =$$

$$\frac{1}{\lambda}\begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0 \cos H_0 & \sin\delta_0 \sin H_0 & \cos\delta_0 \\ \cos\delta_0\cos H_0 & -\cos\delta_0 \sin H_0 & \sin\delta_0 \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Full derivation in Appendix

The visibility function

The 2-element interferometer : The visibility function

$$\underline{V} = |V|e^{\imath\phi_V} = \int_\Omega A(\boldsymbol{\sigma}) I_\nu(\boldsymbol{\sigma}) e^{-\imath 2\pi \frac{\mathbf{b}\cdot\boldsymbol{\sigma}}{\lambda}} d\Omega$$

The 2-element interferometer : The visibility function

$$\underline{V} = |V|e^{\imath\phi_V} = \int_\Omega A(\boldsymbol{\sigma}) I_\nu(\boldsymbol{\sigma}) e^{-\imath 2\pi \frac{\mathbf{b}\cdot\boldsymbol{\sigma}}{\lambda}} d\Omega$$

$$\mathbf{s_0} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \boldsymbol{\sigma} = \begin{pmatrix} l \\ m \\ n \end{pmatrix} \quad \mathbf{b}_\lambda = \frac{\mathbf{b}}{\lambda} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

$$\mathbf{b}_\lambda \cdot \boldsymbol{\sigma} = ul + vm + (n-1)w$$

The 2-element interferometer : The visibility function

$$\underline{V} = |V|e^{\imath\phi_V} = \int_\Omega A(\boldsymbol{\sigma}) I_\nu(\boldsymbol{\sigma}) e^{-\imath 2\pi \frac{\mathbf{b}\cdot\boldsymbol{\sigma}}{\lambda}} d\Omega$$

$$\mathbf{s_0} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \boldsymbol{\sigma} = \begin{pmatrix} l \\ m \\ n \end{pmatrix} \quad \mathbf{b}_\lambda = \frac{\mathbf{b}}{\lambda} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

$$\mathbf{b}_\lambda \cdot \boldsymbol{\sigma} = ul + vm + (n-1)w$$

$$d\Omega = \frac{dl\,dm}{n} = \frac{dl\,dm}{\sqrt{1 - l^2 - m^2}}$$

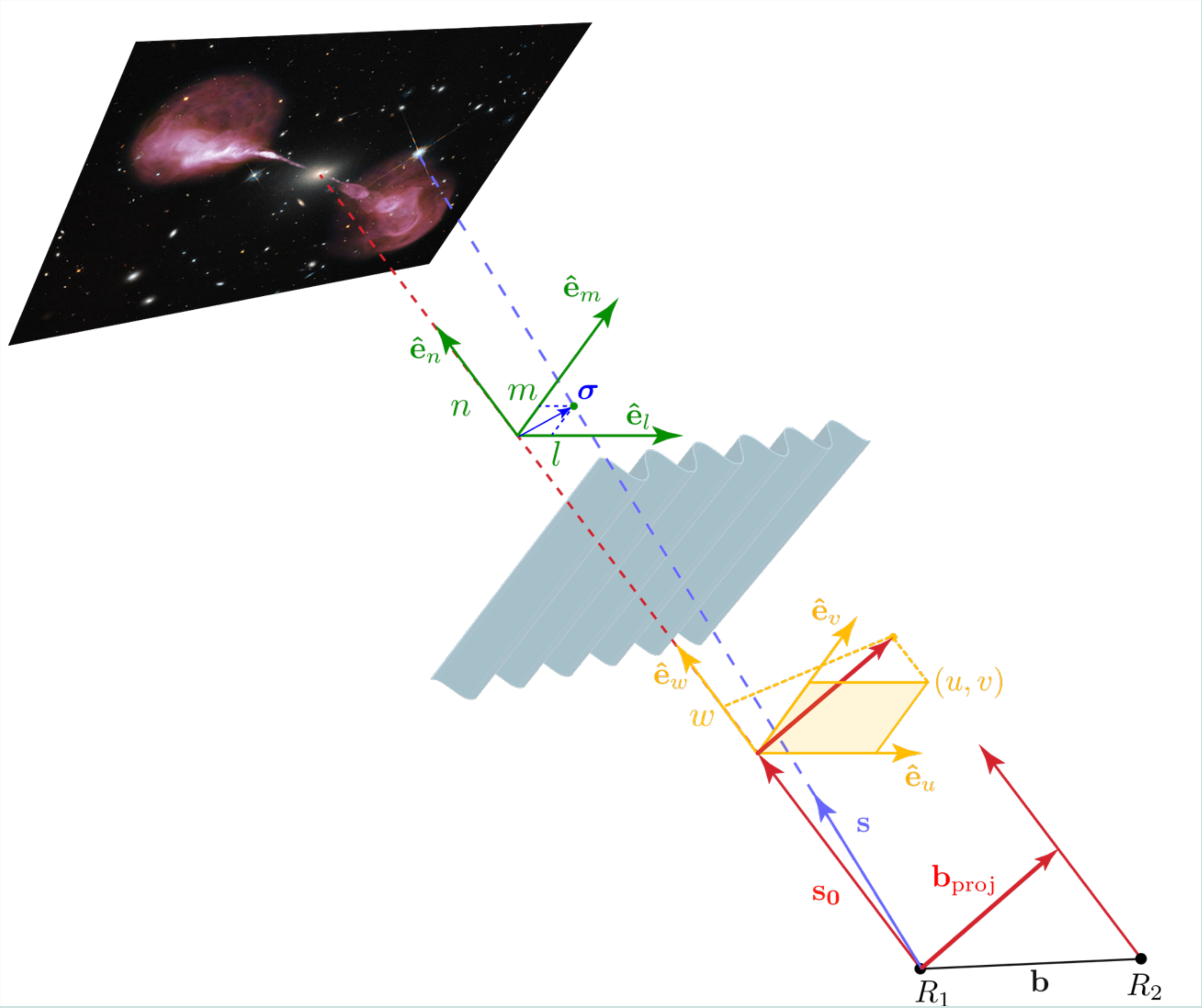
The 2-element interferometer :The visibility function

$$\mathcal{V}(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(l,m) I_\nu(l,m)$$

X

$$\exp\left[-\imath 2\pi(ul + vm + w(\sqrt{1-l^2-m^2}-1))\right]\frac{dldm}{\sqrt{1-l^2-m^2}}$$

The 2-element interferometer : The visibility function



The 2-element interferometer : the Van-Cittert Zernike theorem

$$\mathcal{V}(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(l,m)I_\nu(l,m)\exp\left[-\imath 2\pi(ul+vm+w(\sqrt{1-l^2-m^2}-1))\right]\frac{dldm}{\sqrt{1-l^2-m^2}}$$

Not a FT

The 2-element interferometer : the Van-Cittert Zernike theorem

$$\mathcal{V}(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(l,m) I_\nu(l,m) \exp\left[-\imath 2\pi (ul + vm + w(\sqrt{1-l^2-m^2}-1))\right] \frac{dl dm}{\sqrt{1-l^2-m^2}}$$

Not a FT

Mutual incoherence of the source

Far-field approximation $R >> \frac{|\mathbf{b}_{\max}|^2}{\lambda}$

Homogeneity of the propagation medium

Small field approximation $A(l,m) \approx cte$ $(\alpha,\delta) \leftrightarrow (l,m)$

$w(\sqrt{1-l^2-m^2}-1) \rightarrow 0$ $\Delta\theta_{\rm source} << \Delta\theta_{\rm FoV}$

Co-planar baseline $w \sim 0$

Narrowband approximation $\frac{\Delta\nu}{\nu} < \frac{1}{l_{\max} u}, \frac{1}{m_{\max} v}$

Continuous sampling approximation

The 2-element interferometer : the Van-Cittert Zernike theorem

$$\mathcal{V}(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(l,m)I_\nu(l,m)\exp\left[-\imath 2\pi(ul+vm+w(\sqrt{1-l^2-m^2}-1))\right]\frac{dldm}{\sqrt{1-l^2-m^2}}$$

Not a FT

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Narrowband approximation $\frac{\Delta\nu}{\nu} < \frac{1}{l_{\max}u}, \frac{1}{m_{\max}v}$

Continuous sampling approximation

FT

$$V_{pq}(u,v,0) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} I_\nu(l,m)e^{-2\imath\pi(ul+vm)}dldm$$

The 2-element interferometer : the Van-Cittert Zernike theorem

$$\mathcal{V}(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(l,m) I_\nu(l,m) \exp\left[-\imath 2\pi(ul+vm+w(\sqrt{1-l^2-m^2}-1))\right] \frac{dl dm}{\sqrt{1-l^2-m^2}}$$

Not a FT

Mutual incoherence of the source

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FT

$$V_{pq}(u,v,0) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} I_\nu(l,m) e^{-2\imath\pi(ul+vm)} dl dm$$

$$\mathcal{V}(u,v) \sim \mathcal{F}\{I_\nu\}(u,v)$$

Sampling by an interferometer
UV tracks

The 2-element interferometer : UV tracks

UV plane

2D plane containing the collection of all (u, v) measurements

UV tracks

Trajectory of one projected baseline associated with the same physical baseline.

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0 \cos H_0 & \sin\delta_0 \sin H_0 & \cos\delta_0 \\ \cos\delta_0 \cos H_0 & -\cos\delta_0 \sin H_0 & \sin\delta_0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

with

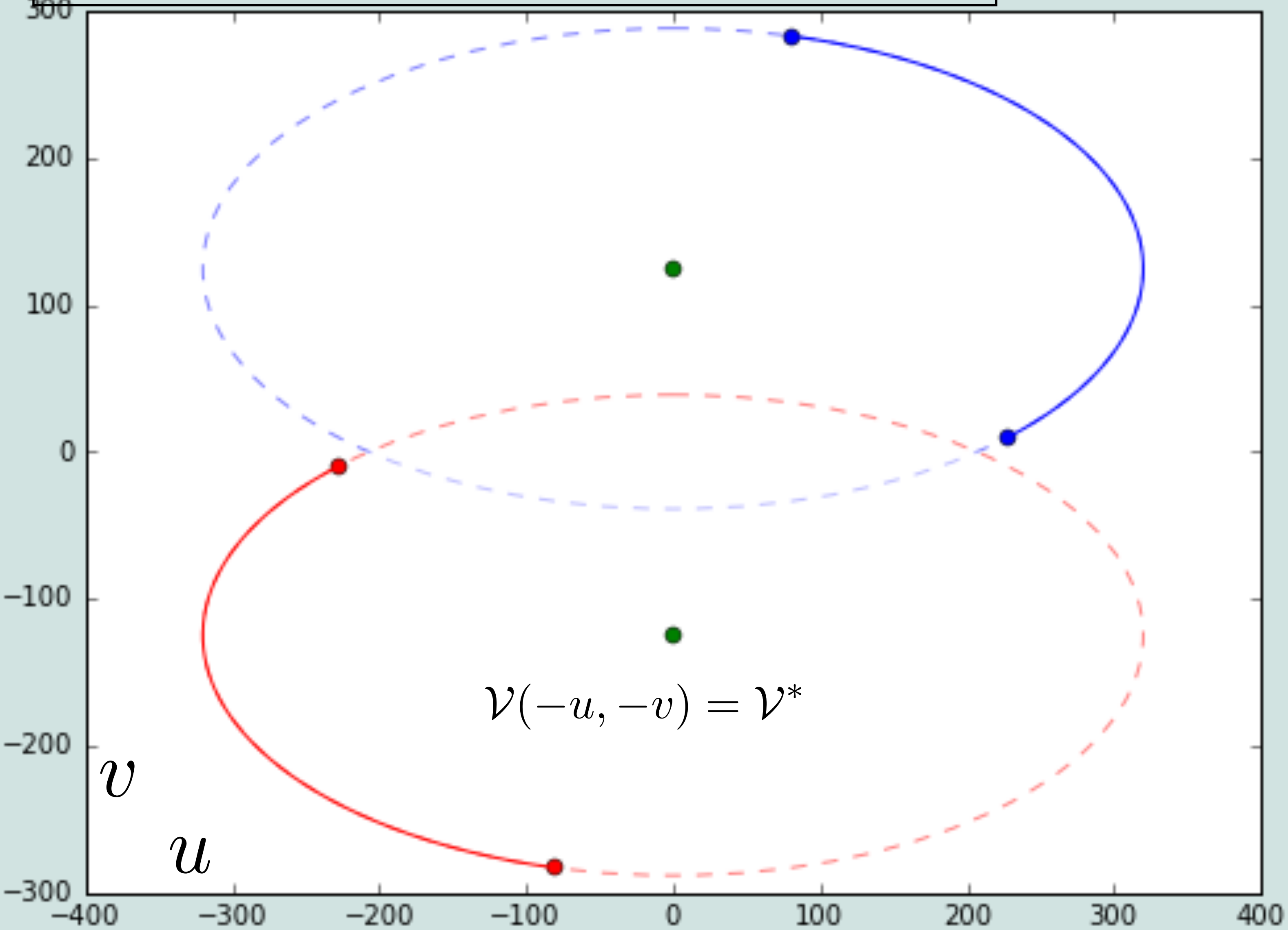
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = |\mathbf{b}| \begin{bmatrix} \cos L_a \sin\mathcal{E} - \sin L_a \cos\mathcal{E} \cos\mathcal{A} \\ \cos\mathcal{E} \sin\mathcal{A} \\ \sin L_a \sin\mathcal{E} + \cos L_a \cos\mathcal{E} \cos\mathcal{A} \end{bmatrix}$$

The 2-element interferometer : UV tracks

$$u^2 + \left[\frac{v - \frac{Z}{\lambda}\cos\delta}{\sin\delta}\right]^2 = \left[\frac{X}{\lambda}\right]^2 + \left[\frac{Y}{\lambda}\right]^2$$

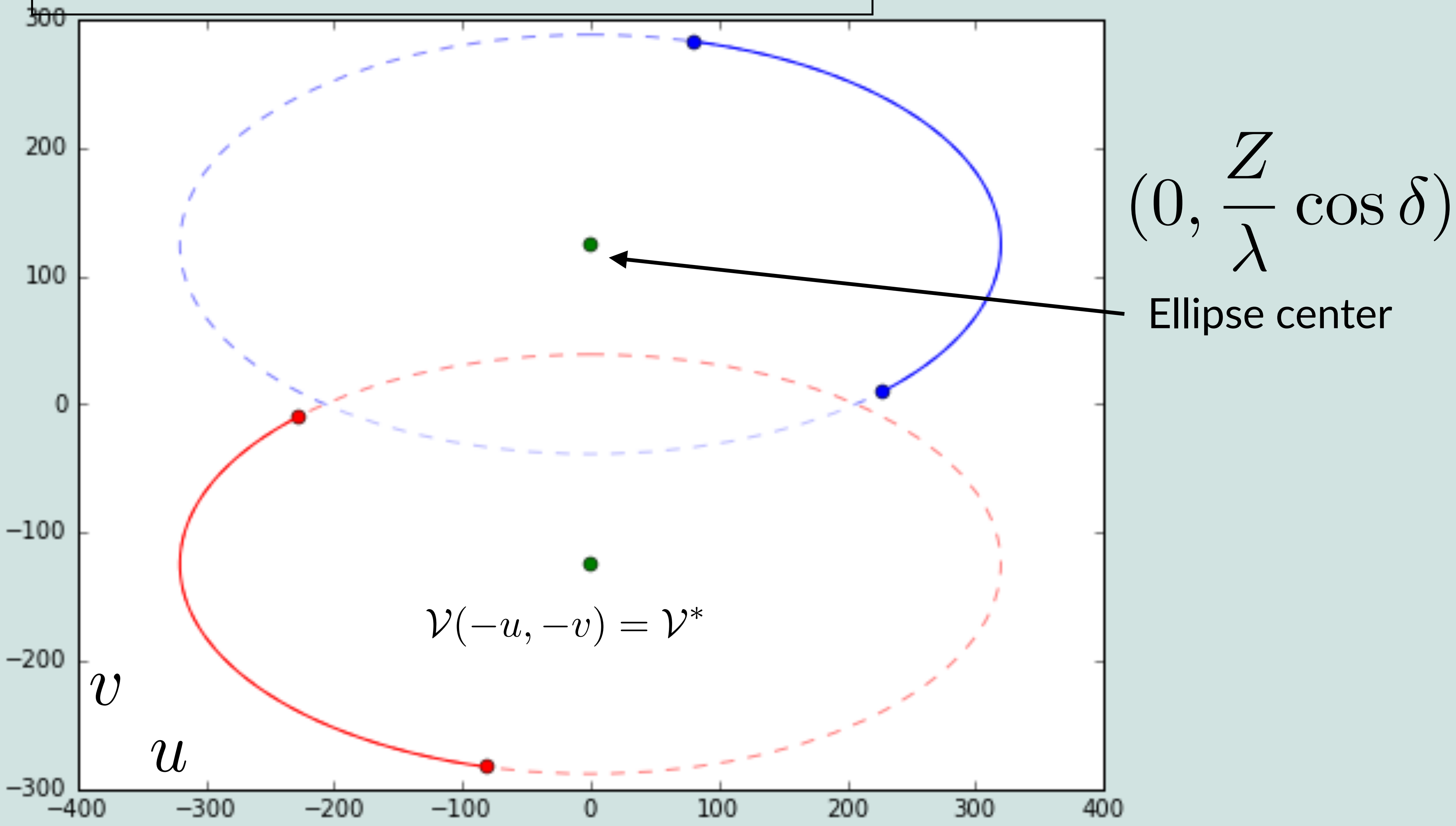
The 2-element interferometer : UV tracks

$$u^2 + \left[\frac{v - \frac{Z}{\lambda}\cos\delta}{\sin\delta}\right]^2 = \left[\frac{X}{\lambda}\right]^2 + \left[\frac{Y}{\lambda}\right]^2$$



The 2-element interferometer : UV tracks

$$u^2 + \left[\frac{v - \frac{Z}{\lambda}\cos\delta}{\sin\delta}\right]^2 = \left[\frac{X}{\lambda}\right]^2 + \left[\frac{Y}{\lambda}\right]^2$$



The 2-element interferometer : UV tracks

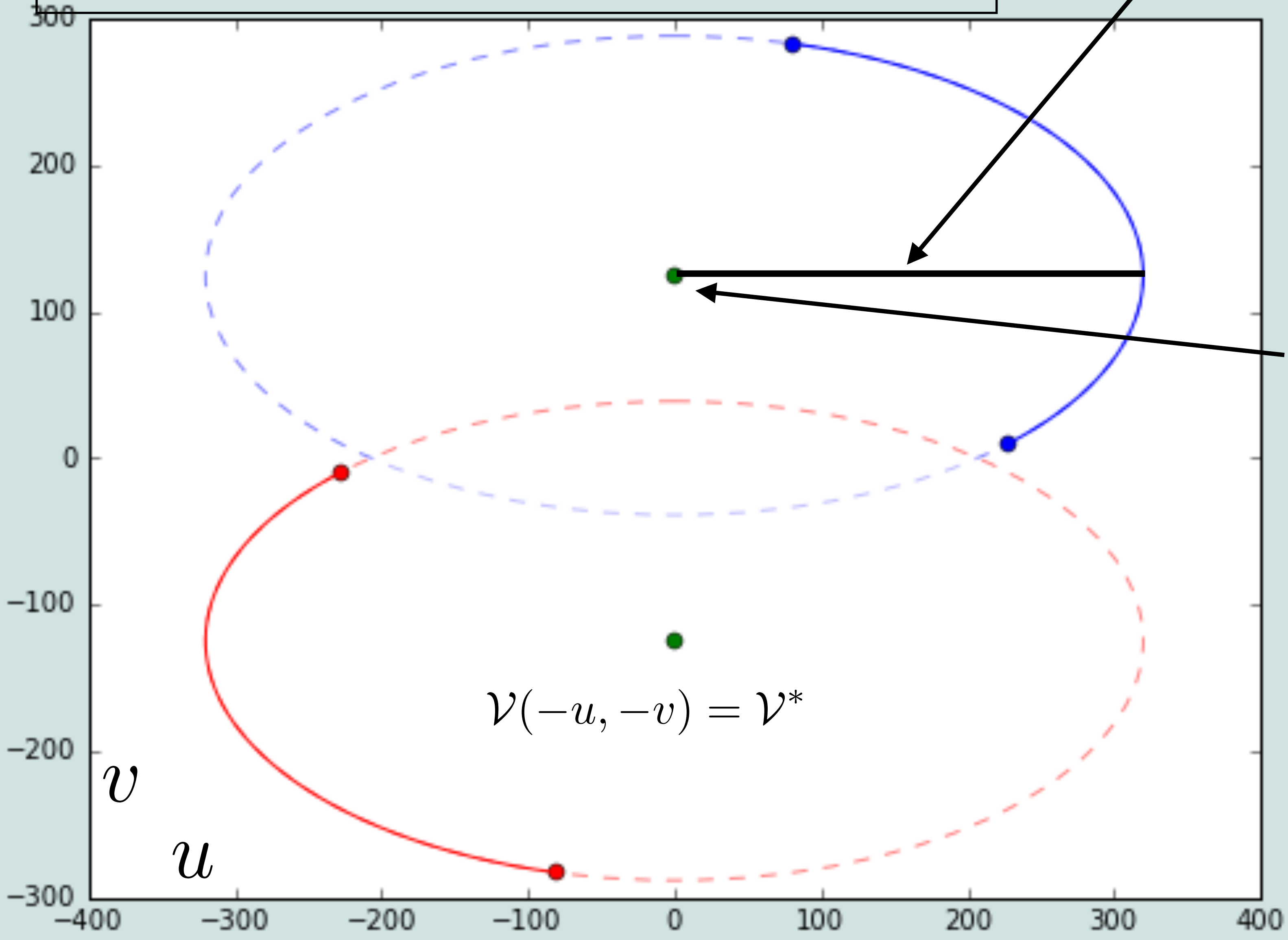
$$u^2 + \left[\frac{v - \frac{Z}{\lambda}\cos\delta}{\sin\delta}\right]^2 = \left[\frac{X}{\lambda}\right]^2 + \left[\frac{Y}{\lambda}\right]^2$$

$$a = \frac{\sqrt{X^2 + Y^2}}{\lambda}$$

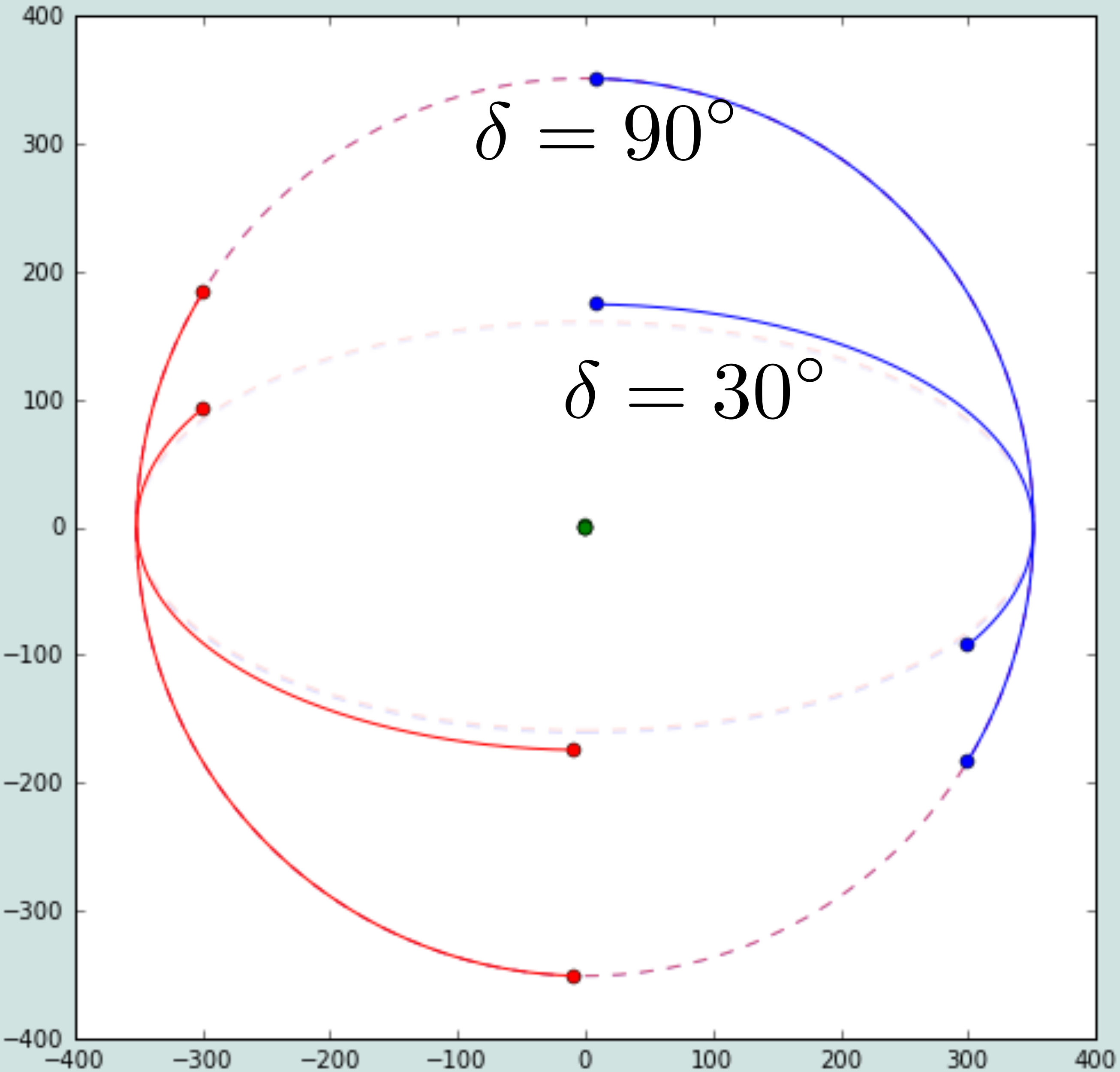
Semi-major axis

$$(0, \frac{Z}{\lambda}\cos\delta)$$

Ellipse center



The 2-element interferometer : UV tracks



Polar array observing

at the NCP

$$\delta = 90^\circ$$

at declination

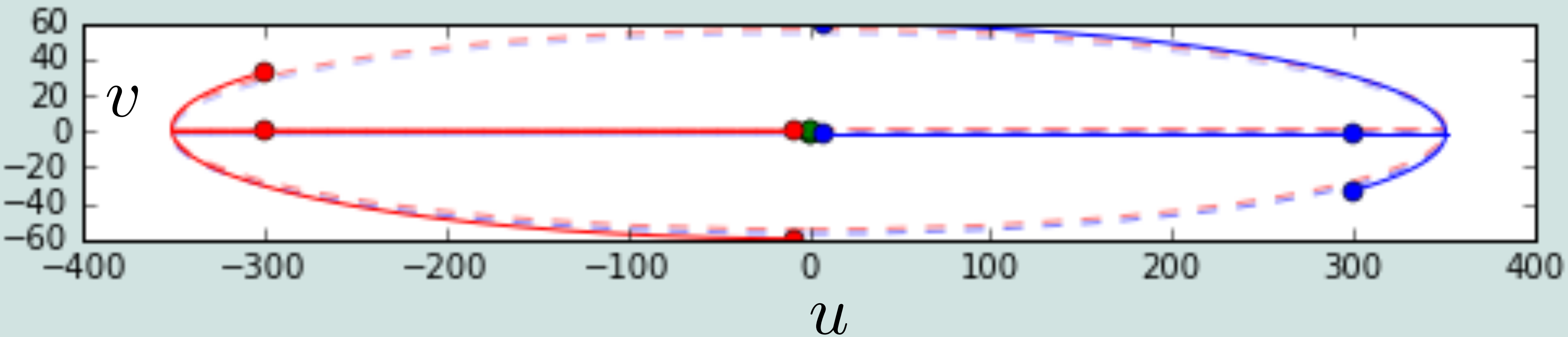
$$\delta = 30^\circ$$

UV tracks are circular when observing at the Pole

The 2-element interferometer : UV tracks

Equatorial array observing at the equator $\delta = 0^\circ$

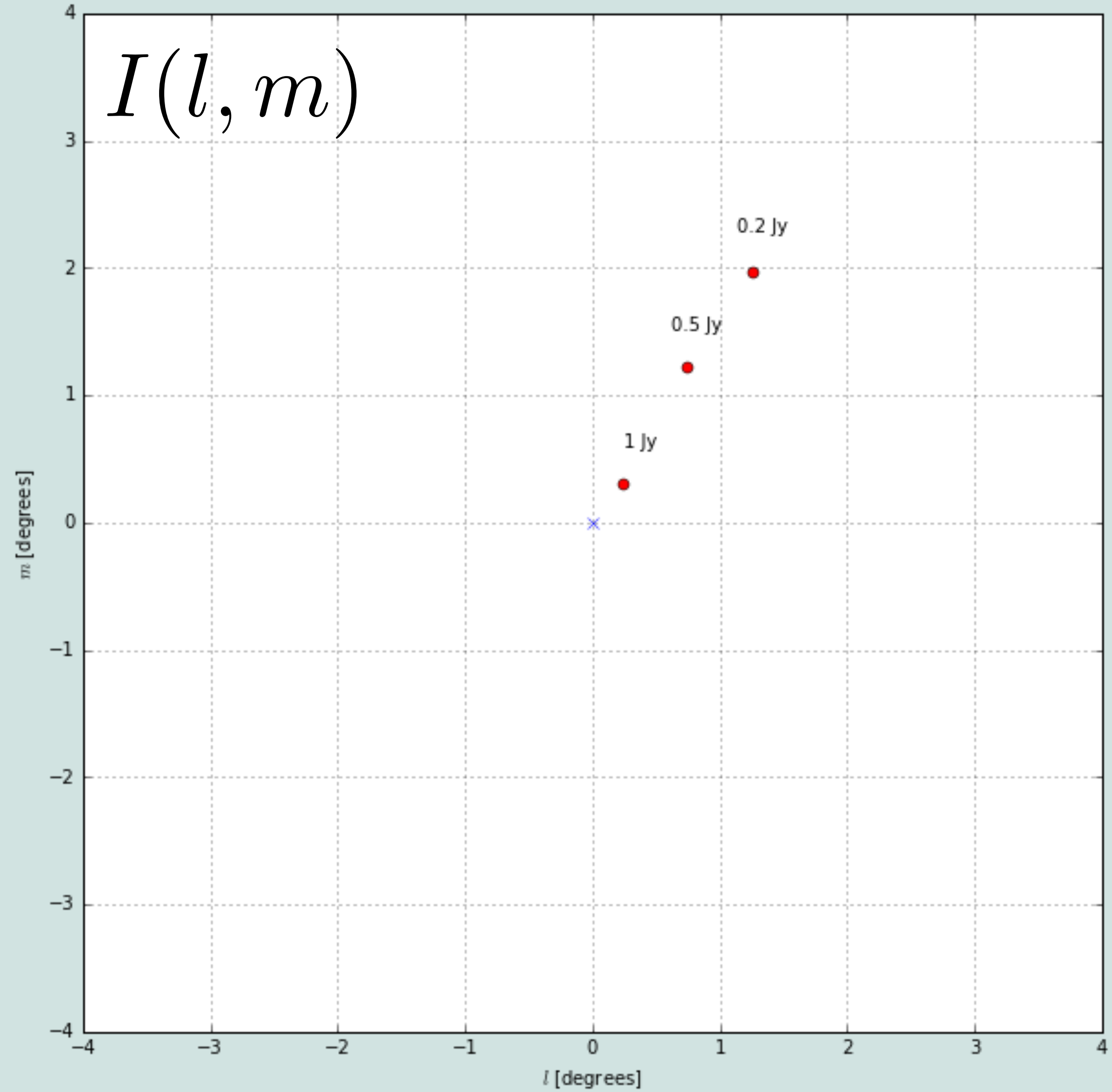
at declination $\delta = 10^\circ$



UV tracks are linear when observing at the equator

The 2-element interferometer : Sampling by an interferometer

Simulating a fake sky



The 2-element interferometer : Sampling by an interferometer

Simulating a fake sky

Phase center

$(\alpha_0, \delta_0) = (\text{5h 30m}, 60°)$

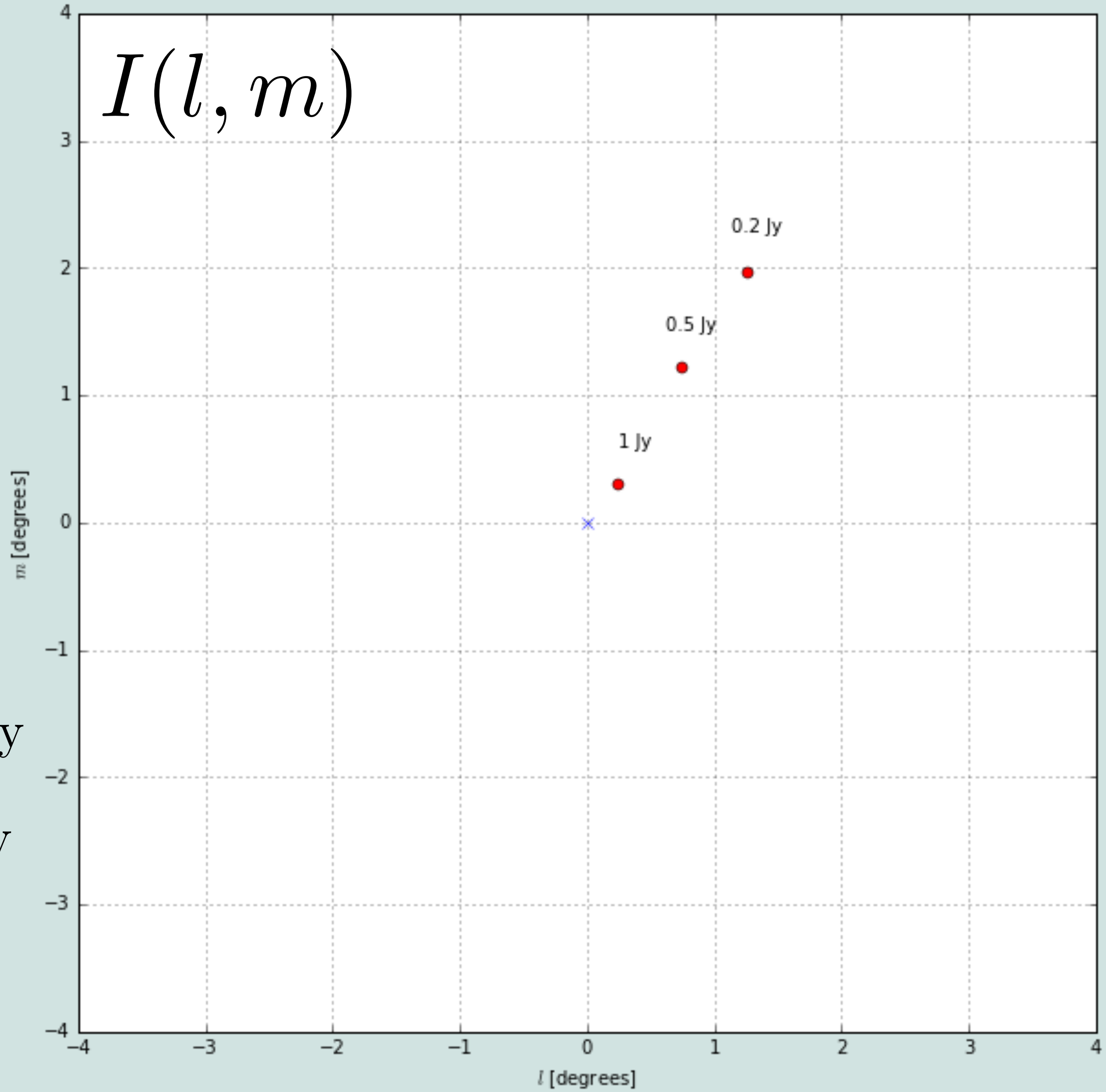
Sources

α δ

(5h 32m 0.4s,60°-17' 57") - 1 Jy

(5h 36m 12.8s,-61° 12' 6.9") - 0.5 Jy

(5h 40m 45.5s,-61° 56' 34") - 0.2 Jy



The 2-element interferometer : Sampling by an interferometer

Simulating a fake sky

Phase center

$(\alpha_0, \delta_0) = $ (5h 30m,60°)

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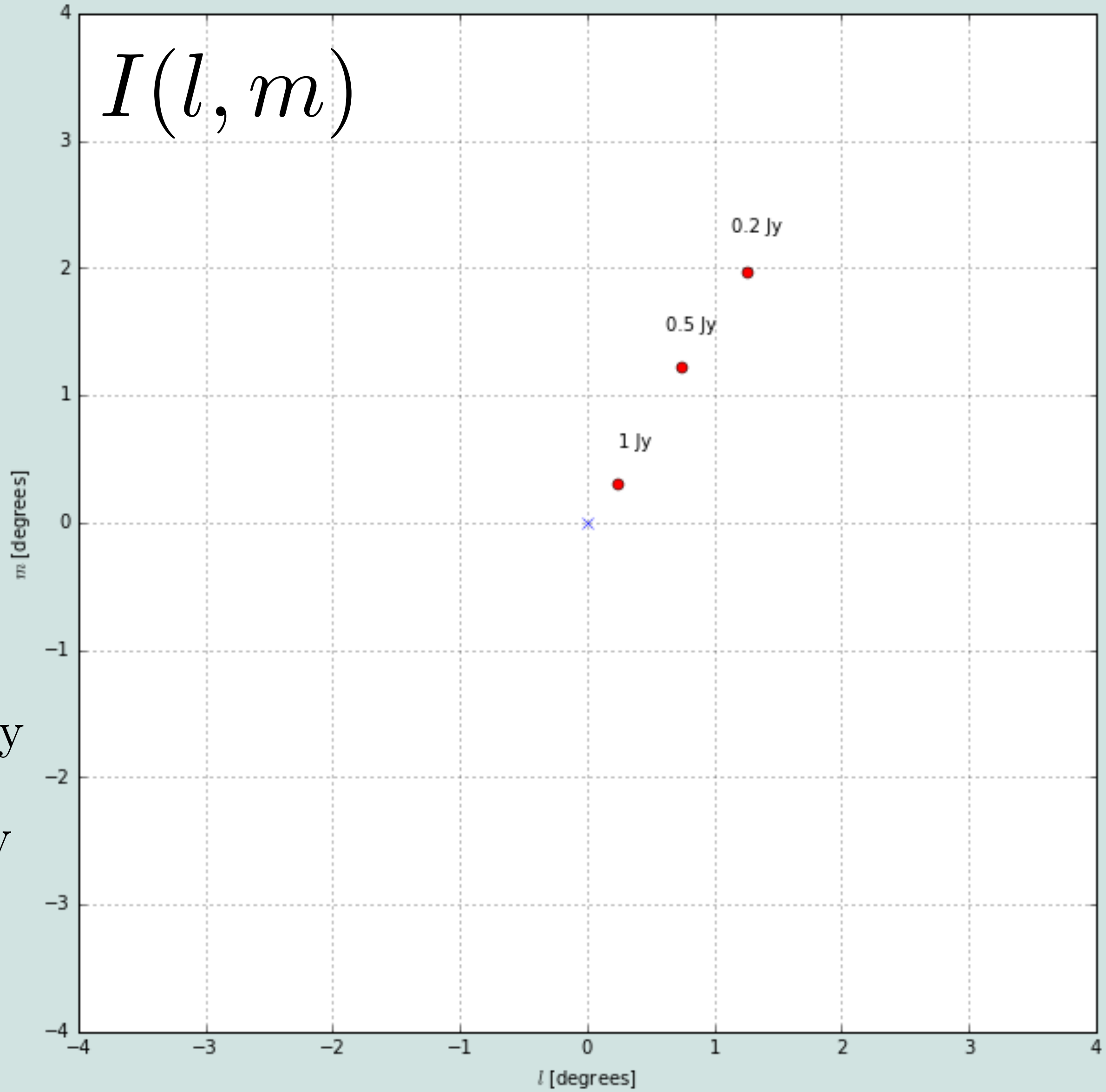


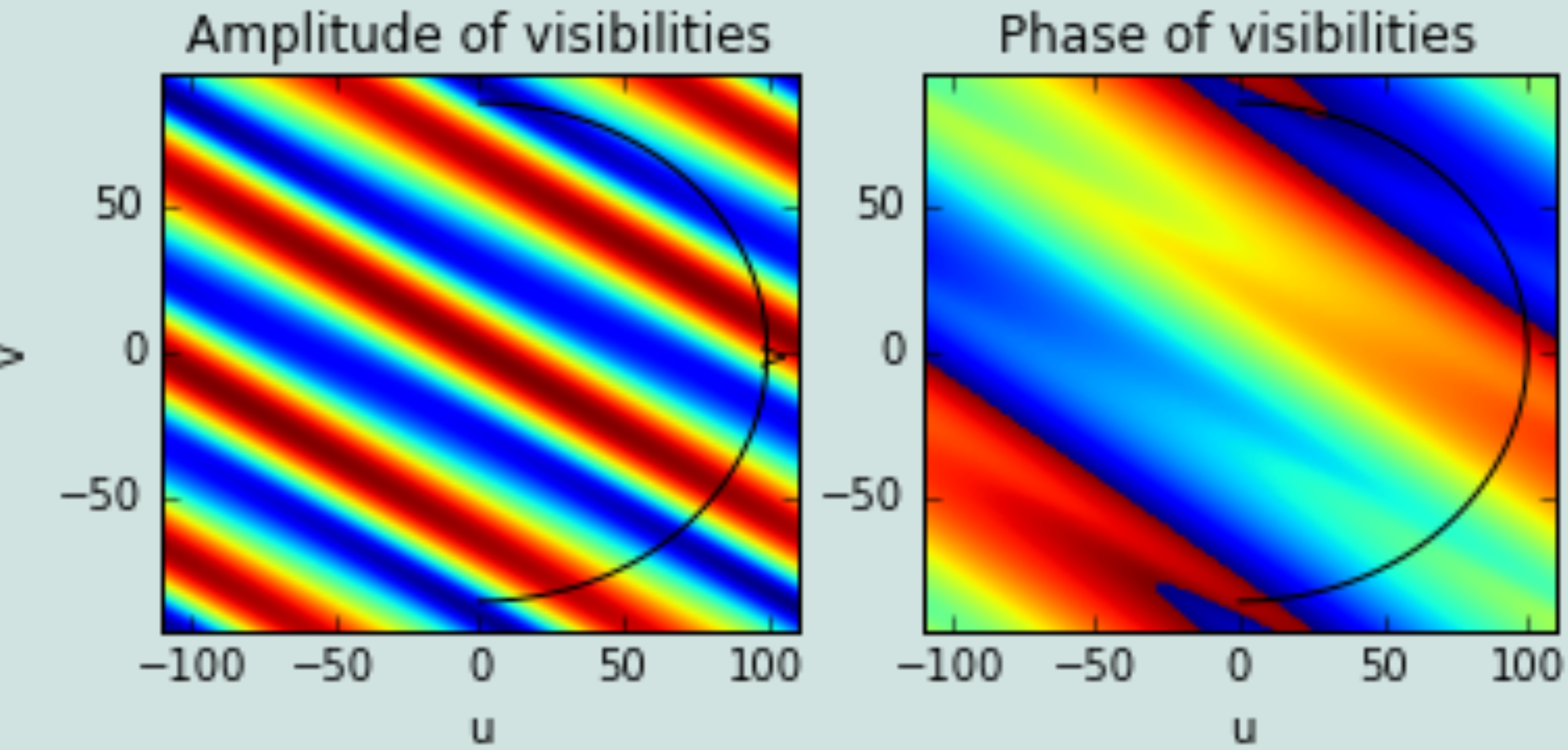
Converting to (l,m)

$$l = \cos\delta \sin\Delta\alpha$$
$$m = \sin\delta\cos\delta_0 - \cos\delta\sin\delta_0\cos\Delta\alpha$$
$$\Delta\alpha = \alpha - \alpha_0$$

The 2-element interferometer : Sampling by an interferometer

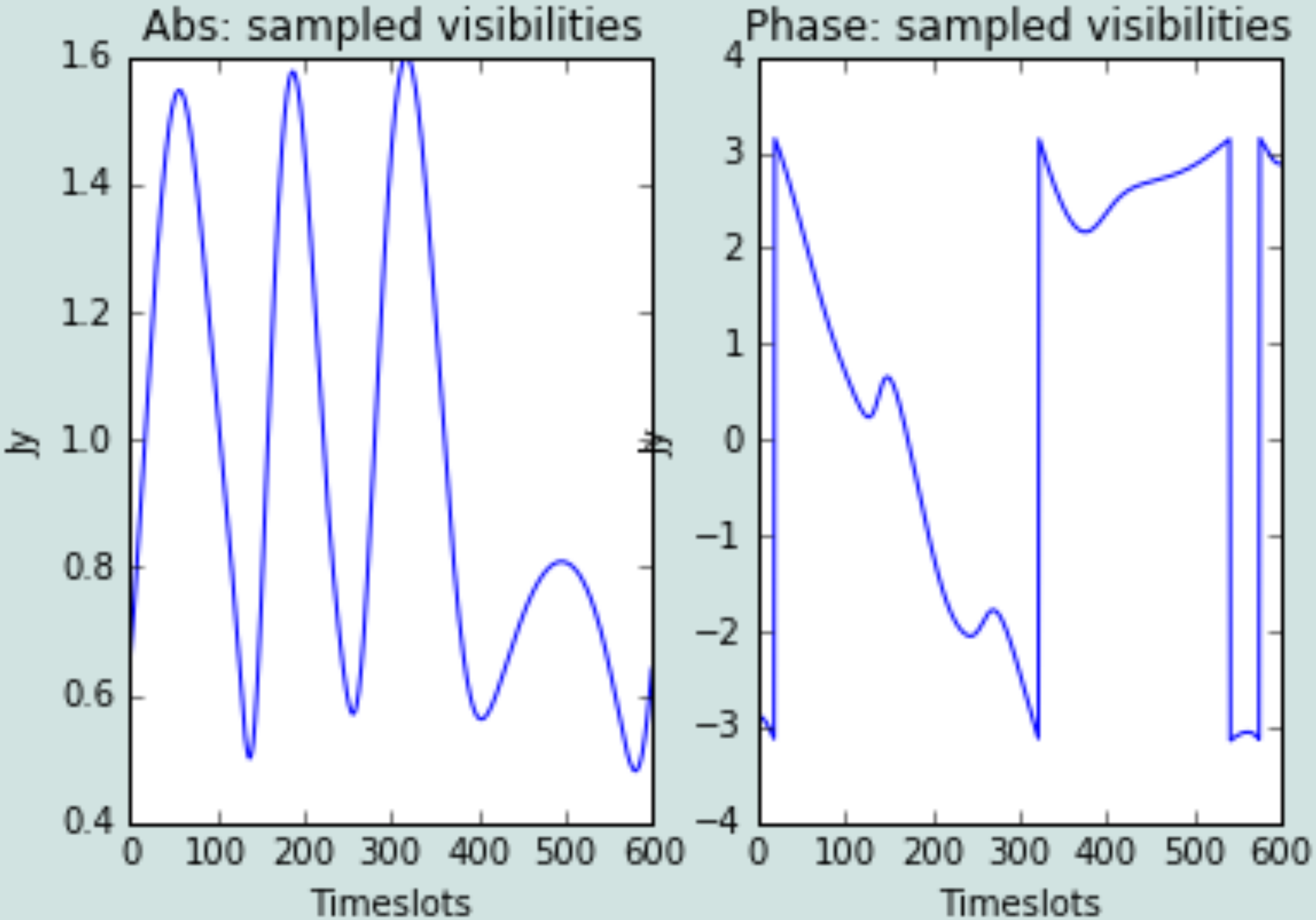
Simulating visibilities $V(u,v)$

$$V(u,v) = \mathcal{F}\{I(l,m)\} = \mathcal{F}\{\sum_k A_k \delta(l-l_k, m-m_k)\} = \sum_k A_k e^{-2\pi i(ul_i + vm_i)}$$



The 2-element interferometer : Sampling by an interferometer

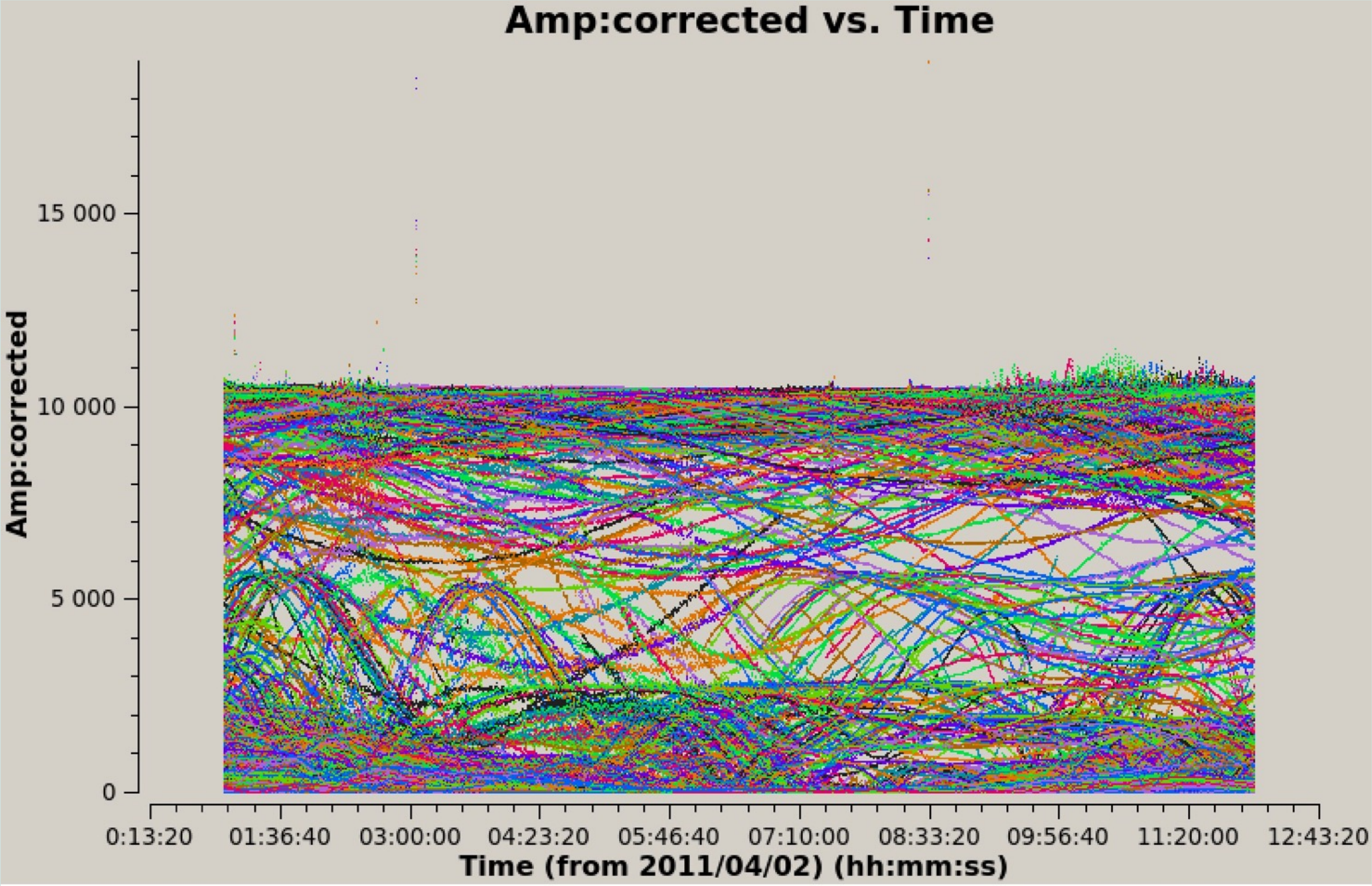
$$V(u,v) = \mathcal{F}\{I(l,m)\} = \mathcal{F}\{\sum_k A_k \delta(l-l_k, m-m_k)\} = \sum_k A_k e^{-2\pi i(ul_i+vm_i}$$



Improving the uv coverage
(if time available)

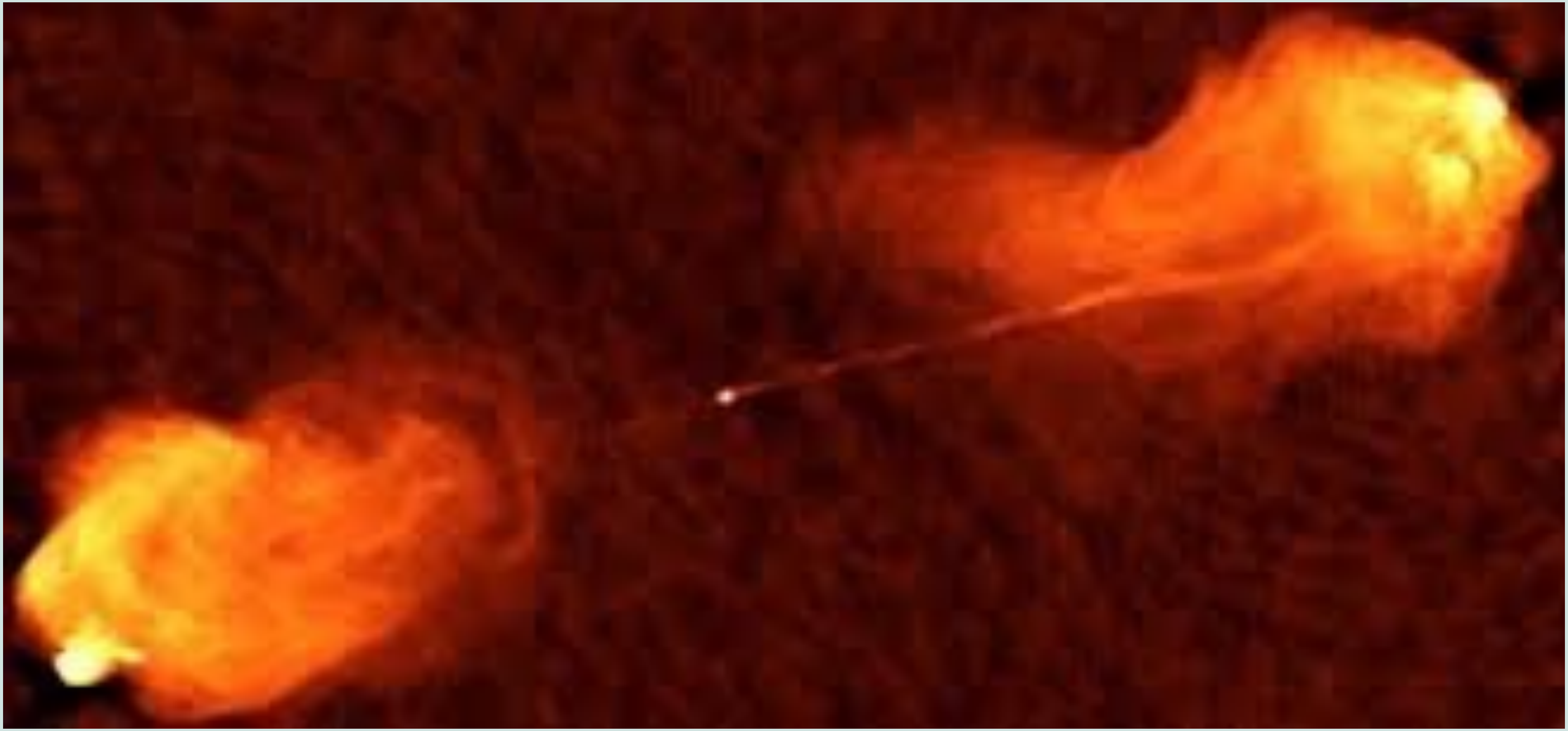
The 2-element interferometer : Sampling by an interferometer

In the next chapter, you will learn, how to turn that



The 2-element interferometer : Sampling by an interferometer

Into that



The 2-element interferometer : The complex correlator

finite bandwidth , sinc, directivity therefore delay tracking

$$d\underline{C} = |V| e^{\imath(\phi_V - 2\pi\nu\tau)} d\nu$$

$$\tau = \frac{\mathbf{b} \cdot \mathbf{s_0}}{c}$$